MODELLING PINUS PINASTER DATA

Introduction

Horne, Robinson and Gwalter (1986) found Response Increment an effective method to analyse thinning trials in even aged forests. The procedure, illustrated for *Pinus radiata*, separated plot increment data into stand segments (strata) of the largest 100, 200 300, etc trees, on the basis of the stem diameter at the beginning of the increment period. Increment was then partitioned into the base growth of the strata within the unthinned control (Ib) and the response increment (Ir) estimated from regression for plots thinned to strata size (100 s ha⁻¹, 200 s ha⁻¹, etc.). Modified Weibull functions (Yang, Kozak and Smith 1978) were fitted to the base and release increments which are summed to give a smoothed total increment (It).

The procedure divided the data into separate increment periods, apparently for convenience to eliminate large thinning alterations in the range of plots during the selected period. Increment was the average periodic annual growth in stand basal was related to this stocking which was and stand density. Stocking referred to as erroneously convenient for operational prescription but is a variable which is only relevant when related to stand age and site quality. that the release authors note for instance The approximating maximum thinning response decreased in stocking This is an obvious with stand age of the increment period. result of stand density of any strata, fixed on a stocking basis, increasing with age. Similarly, the same strata will have increasing stand density with increases in site quality. Analysis could be more meaningful if associated with stand density instead of stand stocking.

Increment would be best based on stand volume rather than stand basal area. For most plot data for which diameter measurement is accurate this is impractical as tree height measurements are sparse and/or of a reduced level of accuracy.

The main point stressed for response analyses by Horne, Robinson and Gwalter (1986) and Horne and Robinson (1988) is that the smoothed total increment curve, It, obtained by adding Ir and Ib approximates Langsaeter's density growth graph which contains a plateau zone where stand increment is independent of density. Horne and Robinson (1988) found that the plateau phenomenon occurred irrespective of location or site potential. However, plateau relationships were not found for young stands, particularly those less than 10 years old.

Horne and Robinson identified the first derivative of the Weibull equation, Ir = f(N), equated to zero and solved for N, as the stocking at which the response increment is maximised. This provides an estimate of the critical point stocking and associated increment on the total increment curve can be calculated. The second derivative of the equation, equated to

zero and solved for N gave the stocking at which the point of inflexion occurs on the Ir curve. This can be used to estimate the increment at this critical point on the total increment curve. These critical points were related to maximum stand productivity at the point of inflexion and optimal piece size at the maximum of Ir.

Horne and Robinson (1988) suggested the utility of the procedure for a wide range of thinning studies. The current analysis links *Pinus pinaster* data from three thinning series through the Response Increment procedure.

Table 1. Periodic annual increment in basal area for stand segments of the largest diameter stems for a range of thinning treatments and the unthinned control. Table values are It for each stand segment (strata) and control values are the base Ib. Response values are obtained by subtracting Ib from It. Data refers to South lane Poole for the period 1958-64.

Mean			Stand	segment	(stems	ha-1)		
Stock (s ha-	-1) 25	50	75	100	200	300	400	500
283	0.134	0.293	0.433	0.555	1.066	*	*	*
408	0.125	0.245	0.358	0.460	0.897	1.220	1.414	*
512	0.132	0.223	0.330	0.436	0.759	1.073	1.369	1.586
764	0.068	0.138	0.204	0.268	0.520	0.766	0.999	1.188
1016	0.094	0.180	0.242	0.303	0.497	0.692	0.865	1.037
2591	0.053	0.112	0.152	0.194	0.369	0.499	0.643	0.735
Mean Stock			Stand s	segment	(stems 1	na-1)		
	-1) 600	700	800	900	1000	1200	Total	Treat
283	*	*	*	*	*	*	1.386	
408	*	*	*	*	*	*	1.509	2
512	*	*	*	*	*	*	1.634	3
764	1.347	1.505	*	*	*	*	1.596	4
1016	1.254	1.415	1.554	1.665	1.721	*	1.775	5
2591	0.859	0.966	1.058	1.140	1.214	1.352	1.834	6(Ib)

The Response Increment Method - Details of the method are documented by Horne, Robinson and Gwalter (1986) and Horne and Robinson (1988). The current treatment outlines important procedural steps.

The basal area increment of residual stocking following is considered as a combination of increments, a time dependent increment and a competition dependent increment. Time dependent (base) increment (Ib) is that accruing to any part or segment of the stand without a reduction in stand stocking i.e. it relates to segments of the largest stems of the unthinned control. Competition dependent (or response) increment (Ir) is the additional increment

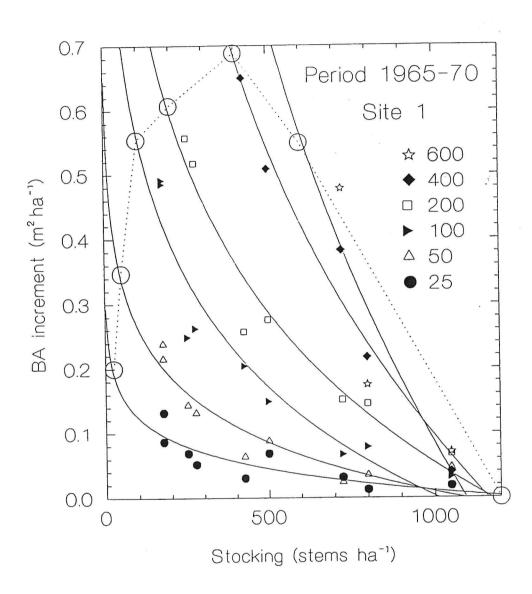


Figure 1. Log curve lines fitted to points of Ir data for strata of different accumulations of the largest trees for each thinning treatment in the Gnangara Basal Area Control series. The large circles are the strata limits determined by regression and the dotted line indicates the trend for the Ir curve to be fitted.

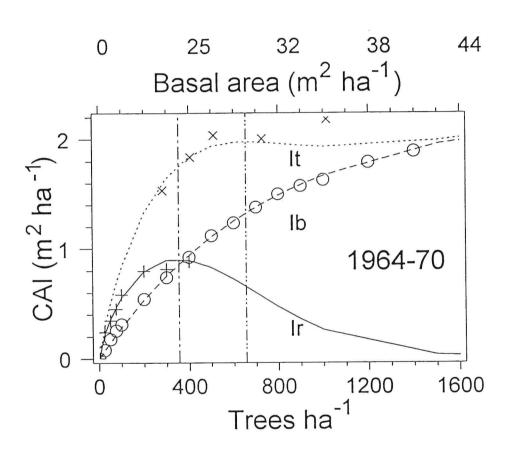


Figure 2. Points and fitted curves for base (Ib) and release (Ir) increments for Late Thinning Trial data. Points for It were original summations for treatments but the dotted line is the sum of Ib and Ir values from fitted curves. The maximum and point of inflexion of the release curve are plotted for relation to the trend of It.

accruing to any part or segment of the stand as a response to a reduction in the stand stocking. The total increment (It) is the sum of these two partial increments thus;

$$It = Ib + Ir \tag{1}$$

Response Increment Analysis concerns stand segments comprising the largest diameter trees (e.g. the largest 100, 200, 300, etc trees ha⁻¹) within a stand stocking of N trees ha⁻¹.

Plot increment, separated into data sets for increment periods and, where relevant, into individual sites, were sorted in descending order on the basis of stem diameter at commencement of the increment period. These sets were summed for the largest 100, 200, 300 to 1200 stems half and the total plot (Table 1). Ib is the value of segments for the unthinned plots.

Columns for strata were copied into a matrix, inverted and the base increment subtracted from each plot strata total to provide values for Ir. The matrix was inverted to provide Ir values for relevant segments in each plot (thinning level), plus Ib and It. Thus data sets were compiled for the stratum of 100 trees ha⁻¹ within a variety of stand stockings representing the trial thinning levels. The maximum Ir value for a particular stand segment (e.g. for S equals 100 trees ha⁻¹) occurs where the stand segment becomes the residual stand stocking (Plot). These were determined from the following logarithmic relationship (Fig. 1):

$$Ir = b_O - b_1 loge N$$
 (2)

The equation was solved and used where the regression coefficient was shown to be significant at the .05 level.

Curves from a modified Weibull function were fitted as follows;

Ib =
$$a[1 - exp{-(N/b)^C}]$$
 (3)

Ir =
$$a(c/b)(N/b)^{c-1}exp\{-(N/b)^{c}\}$$
 (4)

where N = stand segment stocking and a, b and c are constants.

For each Ir curve the roots where the first derivative (maximum) and second derivatives (point of inflexion) equalled zero were estimated as reference stocking levels to compare curves from different site groups and increment periods (Fig. 2).

For initial analysis a curve fitting program was unavailable and curves were fitted by eye. When a curve fitting program became available curves were automatically fitted. To do this most suitably a the above equations were modified (Yang, Kozac and Smith, 1978) as follows;

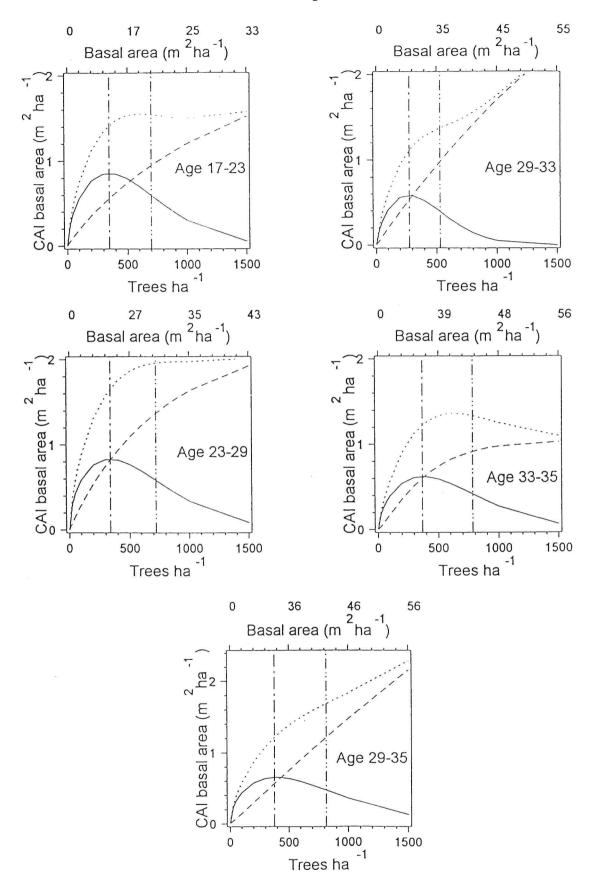


Figure 3. Weibull plots for base increment (Ib - dashed line), release increment(Ir - solid line) and total increment (It - dotted line) for the Late Numerical Thinning Plots. Analyses for a uniform site group are compared for the 1958-64, 1964-70, 1970-74, 1974-76 and 1970-76 increment periods. Increment periods consist of the 20 plots planted in 1941. Stocking and stand density are shown on the lower and upper axis, respectively, of each graph. The maximum and point of inflexion for the Ir curve are indicated by vertical lines.

Table 2. Stocking (s ha⁻¹), stand density (m² ha⁻¹) and total basal area increment (m² ha⁻¹ yr⁻¹) corresponding to the maximum and point of inflexion of the release curve (Ir) for successive strata of the largest trees in thinning trials at Gnangara. Brackets enclose the percentage that the critical point is of the increment at full stocking.

Trial	Age S	Site	I	r max	kimun	ı	Ir	infle	ı To	Total	
	(yrs.)	5	Stock	BA	Inc	%	Stock	BA	Inc	%	Inc
Late Thinning Series	17-23 23-29 29-35	1 1 1	340	15.0 24.4 33.9	1.7		721		2.0	(101) (94) (64)	
	14-18	1 2 3 4 5	439 398	8.7	1.3 1.1		847 738 1015	13.3	1.4 1.4 1.5	(87) (100) (82) (99) (102)	1.8 1.4 1.7 1.6
Free Growth Series	18-24	1 2 3 4 5		17.2 14.6 12.1 9.6 9.1	0.9 0.7	(91) (80) (61) (72) (63)	722 700 549	22.5 21.2 18.3 13.3 13.5	1.2 0.9 0.9	(100) (103) (80) (80) (81)	1.4 1.1 1.2 1.1
	24-32	1 2 3 4 5	215 317 415	24.7 17.9 16.6 16.2 13.9	0.6 0.7 0.7	(75) (47) (73) (75) (73)	473 586 788	30.3 23.1 22.1 22.4 19.2	0.8	(89) (71) (80) (91) (90)	1.2 1.3 1.0 0.9
	19-24		219	13.7 11.4 10.7	1.1	(71) (64) (79)	441	22.3 16.5 26.1	1.3	(83) (73) (87)	1.8
Basal	24-28		202	12.9		(57) (80) (69)	410	17.1 18.9 13.9	1.3	(71) (90) (76)	
Area Control Series	28-33		185	15.2 11.3 11.1	1.1		401	23.1 23.0 16.5	1.3	(59) (69) (68)	
	33-39		164	15.7 16.8 15.7	1.3	(69)	323	24.8 23.5 22.3	1.5	(58) (74) (70)	2.0

Ib =
$$a[1 - exp{-b(N)^C}]$$
 (3)

$$Ir = abc(N)^{C-1}exp\{-b(N)^{C}\}$$
 (4)

where N = stand segment stocking and a, b and c are constants.

Procedure

Wp 15/57 - The Late thinning Series - This trial compares the growth of six treatments of 280, 400, 486, 700 1000 and 2500 (control) s ha⁻¹ in a 1941 stand at Gnangara plantation. original stand was planted at 2 x 2 m spacing and unthinned to time of trial establishment in 1957. The series consisted of 20 plots in the 1941 planting and 5 plots in an adjoining 1942 planting. Analysis was carried out for increment periods of 1958-1964, 1964-70, 1970-74, 1974-76 and 1970-76. The original idea was to cover six year increment periods which were convenient to the measurement data. Plots 20-25 were altered in 1974 and the breakdown to the 1970-74 and 1970-76 increment periods appeared to be required, the later interval only covering the 20 even aged plots. From initial analysis, results were not obviously different for plots 1-20 or 1-25. Final analysis concentrated on the 20 plots of even age for three six year increment periods. The plots had been selected to represent the highest site quality in the 1941 stand and subsequent measurements have indicated that there was no basis separating site qualities between treatment Individual plot data was averaged on the basis of the six treatments to obtain Ib, Ir and It values.

Results are depicted in Figure 3 and Table 2. Basal areas equivalent to the stocking (stratum) levels are included in the graphs and Table to associate the curves with stand density.

WP 16/58 - The Free Growth Trial - A Free Growth Trial was established in 1958 at Gnangara in 1953 planting. Fixed stockings of 1880, 1320, 1270, 740, 450 and 247 s ha⁻¹ were progressively established to avoid the onset of mutual competition within each final stocking. The final thinning adjustment was at age 17 years in 1970. The trial was established as a continuous series to embrace the full range of site classes in the area. Increments were analysed for the 1967-71, 1971-1977 and 1977-85 measurement intervals and separated for 5 site class intervals. Individual plot data was averaged on the basis of the original 7 treatment classes to obtain Ib, It and Ir values.

Results are depicted in Figures 4 and 5 and Table 2.

WP 20/65 - The Basal Area Control Series - This trial was established in 1965 at Gnangara Plantation in 19 year old stands to cover fixed density treatments of 37, 25, 16, 11 and 7 m² ha⁻¹ in 5 site uniformity blocks. Basal area limits in treatments were maintained by regular measurement and thinning.

Increments were analysed for the 1965-70, 1970-74, 1974-79 and 1979-85 measurement periods. As a result of the basal area

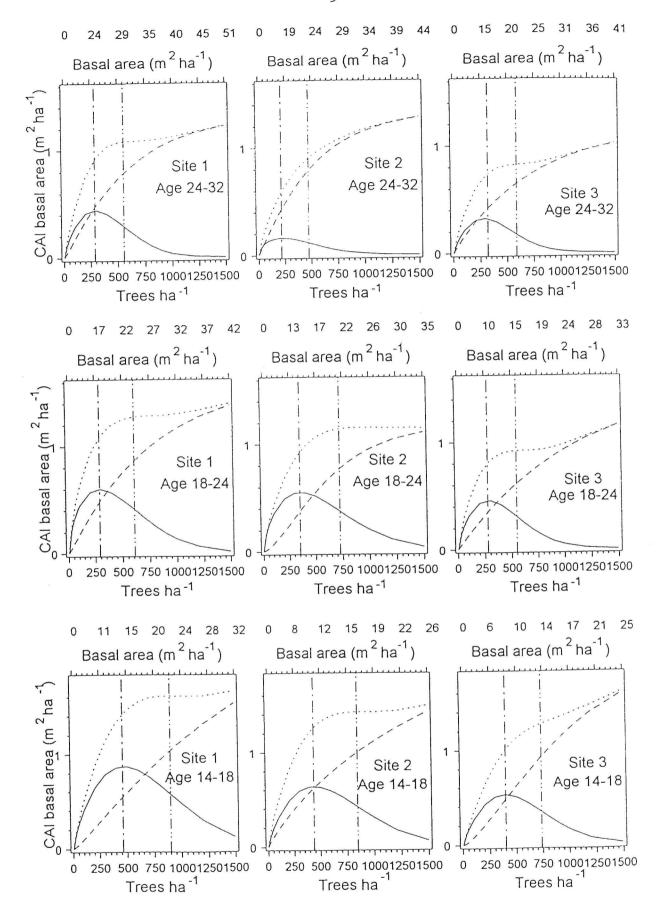


Figure 4. Weibull plots for base increment (Ib - dashed line), release increment (Ir - solid line) and total increment (It - dotted line) for the Free Growth Thinning Plots. Analyses for site groups 1, 2 and 3 are compared for the 1967-71, 1971-77 and 1977-85 increment periods. Stocking and stand density are shown on the lower and upper axis, respectively, of each graph. The maximum and point of inflexion for the Ir curve are indicated by vertical lines.

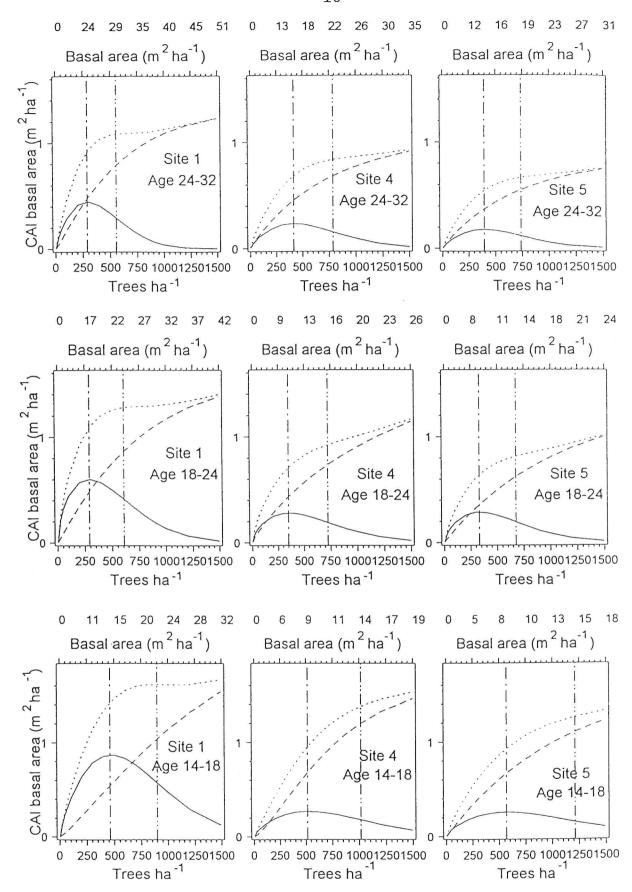


Figure 5. Weibull plots for base increment (Ib - dashed line), release increment (Ir - solid line) and total increment (It - dotted line) for the Free Growth Thinning Plots. Analyses for site groups 1, 4 and 5 are compared for the 1967-71, 1971-77 and 1977-85 increment periods. Stocking and stand density are shown on the lower and upper axis, respectively, of each graph. The maximum and point of inflexion for the Ir curve are indicated by vertical lines.

limit of 37 m^2 , the stocking of the control (base) plots was low in the later stages of measurement. Insufficient plotting points were available for strata in the highest site qualities (high stand density and lower stocking numbers) and for analysis site blocks 1 and 2 and 3 and 4 were combined. The poorest site 5 with highest stockings plotted satisfactorily.

Results are presented in Figures 6 and 7 and Table 2.

Discussion

The Plateau - The main point stressed for response analyses by Horne, Robinson and Gwalter (1986) and Horne and Robinson (1988) is that the smoothed total increment curve It, obtained by adding Ir and Ib, approximates Langsaeter's density growth graph which contains a plateau zone where stand increment is independent of density. Langsaeter plotted volume increment against standing volume, a direct association with stand density, whereas response increment relates basal area increment against stocking which is often a poor indication of stand density for varying growth periods and site conditions.

The present study for *P. pinaster* shows that Response Increment curves can be fitted satisfactorily and often indicate a plateau situation (Figs. 1-7). Figure 2 also shows that it is simple to obtain the total increment curve It from a direct summation of treatment plots.

A problem of general application for the procedure seen from the current data is that for the analyses to be comparative over a range of trials the Control, which provides the base value, must be representative of full stocking at all ages. This was so for the Late Thinning Trial and the Free Growth Trial but not necessarily so for the Basal Area Control Trial. The latter, based on stand density rather than stocking, used a stand density of 37 \rm{m}^2 \rm{ha}^{-1} as the control or limit. This is by no means the highest stand density attainable with age on the sites in question. Stocking numbers for this limit in the later stages of the rotation were too low (at least on the highest site qualities) to confidently represent full stocking. This restricted the Response analysis as too few strata were available for individual site groups to obtain sound data for curve fitting. For this reason sites 1 and 2 and 3 and 4 were combined for analysis (Figs. 6 and 7). At Yanchep, the second part of the Basal area Control Series used a maximum basal area class of 25 m^2 ha⁻¹ as this was the highest reasonable for the area with the knowledge of drought mortality at high stand densities. This trial was not analysed by Response analysis as Ir values are not considered to be adequate at later stand Too few strata remained for useful curve fitting with could continued stand development. Total increment, It, however, be readily obtained and Ir could be estimated by subtraction of the artificial Ib values.

Horne and Robinson (1988) found that the plateau phenomenon (Fig. 2) occurred irrespective of location or site potential. However, plateau relationships were not found for young stands,

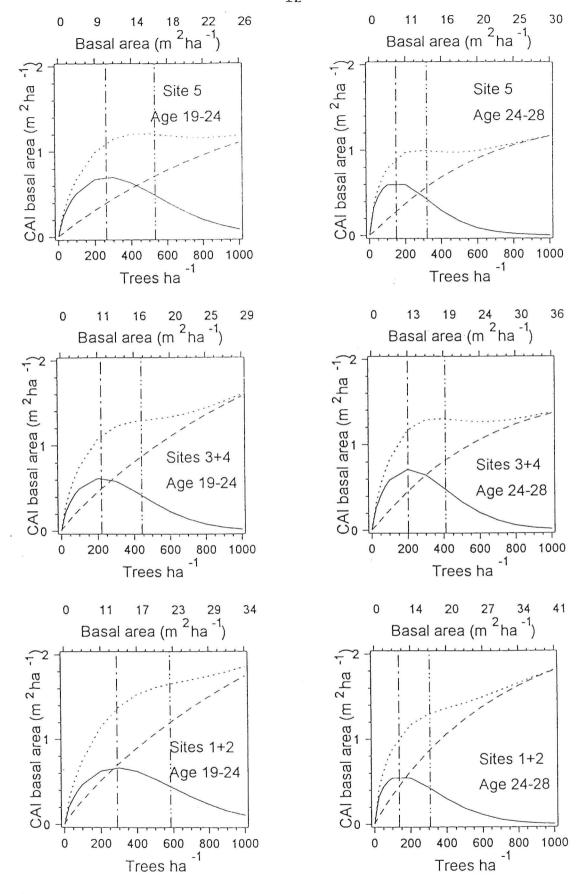


Figure 6. Weibull plots for base increment (Ib - dashed line), release increment (Ir - solid line) and total increment (It - dotted line) for the Basal Area Control thinning Plots. The data are analysed for three site groups for the 1965-70 and 1970-74 increment periods. Stocking and stand density are shown on the lower and upper axis, respectively, of each graph. The maximum and point of inflexion for the Ir curve are indicated by vertical lines.

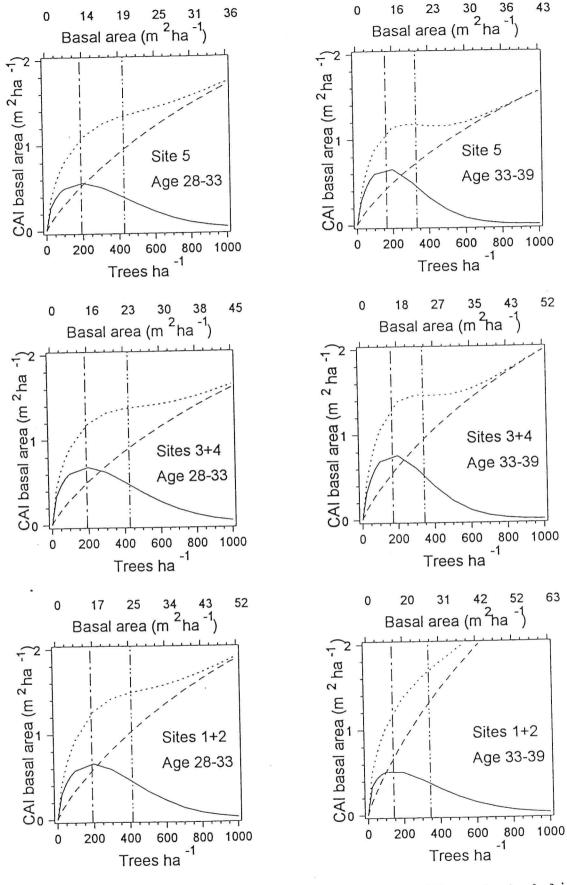


Figure 7. Weibull plots for base increment (Ib - dashed line), release increment (Ir - solid line) and total increment (It - dotted line) for the Basal Area Control thinning Plots. The data are analysed for three site groups for the 1974-79 and 1979-85 increment periods. Stocking and stand density are shown on the lower and upper axis, respectively, of each graph. The maximum and point of inflexion for the Ir curve are indicated by vertical lines.

particularly those less than 10 years old. In fact their data show a plateau was found in 19 of 32 cases studied (59%). The present study also identifies 18 of 30 cases (60%) in which a plateau may be assumed but amongst the exceptions are some of the highest site qualities (Figs. 4, 6, 7). It is also apparent that the choice of the increment period may have some influence on whether or not a plateau is identified in the total increment curve. In Figure 3 a plateau is absent from the 29-33 and 29-35 year old stands but is present within the 33-35 year stand.

In all the *P. pinaster* trials it has been clearly shown that volume increment decreased significantly with reduction of stocking (and stand density) below about 1000 - 1500 stems halt: the Moller plateau occurring with stockings above this limit. These levels do not influence conventional plantation silviculture with the species which uses stockings of 1500 s half or less.

The Late Thinning Trial, in very high quality pine, tended to lose the plateau effect with age (Fig. 3) while for the Free Growth trial (Fig. 8) and Basal Area control Trial (Fig. 9) the slope characteristics tended to be constant with age. Pinus pinaster data, within the range of stocking studied (i.e. for Basal Area Control), the plateau trend can be accepted. It is to be realised however, that a significant decline in increment probably occurs over the range of stockings represented by the flattened increment trend. Rather than define the plateau as a region where increment is more or less constant it is more useful to consider it as a region where increment changes are minimal or zero with increasing stocking.

Critical Points - Horne and Robinson identified the first derivative of the Weibull equation, Ir = f(N), equated to zero and solved for N, as the stocking at which the response increment is maximised. This provides an estimate of the critical stocking, hence the critical increment on the total increment curve can be calculated. Additionally the second derivative of the equation, equated to zero and solved for N will give the stocking at which the point of inflexion occurs on the Ir curve and can be used to estimate the increment of this critical point on the total increment curve. These critical points were related to maximum stand productivity (point of inflexion) and optimal piece size at stocking for the maximum of Ir.

Site Variation - Horne and Robinson noted that in general the basal area increment of the plateau and the stocking at which it commenced were found to decrease with increasing age and declining site potential. The current study cannot support the supposition that for the critical points stocking decreases consistently with site quality. In Figures 8 and 9 there is no consistency in the fluctuations of stocking with site. The curves and summary in Table 2 indicate, however, that the range of stockings covering all sites for each critical point is small and it is not worth separating critical stocking on a site basis. This is not the case for stand densities however,

where for the Free Growth Trial (Table 2, Figs. 4, 5 and 9) stand density in the poorest site 5 is half that of the top site 1, for each of the increment periods studied. For the basal area series (Table 2, Figs. 6, 7, and 9) there are lower values for stand density and increment with decreasing sites but differences over the range of sites grouped are lesser.

Table 3. Parameters for fitting Weibull curve approximations to thinning studies at Gnangara. Release and base curves are for strata containing the largest diameter trees in classes from the unthinned control to the heaviest thinned plot.

Trial	Age	Site		Ir(Relea	ase)	Ib (Base)			
	(years)		a	b(E-5)	С	a	b(E-3)	С	
Late	17-23	1	712	3.0	1.62	2.30	1.2	0.93	
Thinning	23-29	1	755	5.0	1.53	2.26	1.7	0.96	
Series	29-35	1	689	4.0	1.52	9.21	0.1	1.04	
	12-16	1	840	1.0	1.75	2.38	0.2	1.17	
		2	581	1.0	1.75	2.45	1.3	0.89	
		3	411	1.0	1.79	2.00	0.1	1.35	
		4	302	1.0	1.70	1.65	0.2	1.31	
		5	329	1.0	1.67	1.68	1.0	0.98	
	16-22	1	533	5.0	1.60	1.59	1.0	1.05	
Free		2	474	3.0	1.59	1.17	0.1	1.41	
Growth		3	232	3.0	1.62	1.59	1.0	0.98	
Series		4	278	3.0	1.69	1.85	1.5	0.88	
		5	219	3.0	1.63	1.36	1.0	0.99	
	22-30	1	263	2.0	1.76	1.31	0.9	1.10	
		2	96	10.0	1.53	1.35	1.3	1.06	
		3	181	2.0	1.78	1.19	1.6	0.97	
		4	195	1.0	1.77	1.10	1.8	0.94	
		5	137	1.0	1.79	0.88	2.0	0.94	
	19-24	1+2	446	4.0	1.64	3.72	1.3	0.90	
		3+4	313	6.0	1.64	3.40	1.2	0.91	
		5	438	4.0	1.64	1.87	1.0	0.99	
	24-28	1+2	237	22.0	1.48	2.36	1.9	0.96	
Basal		3+4	330	6.0	1.65	1.63	1.2	1.06	
Area		5	242	16.0	1.54	1.38	0.9	1.11	
Control Series	28-33	1+2	255	9.0	1.63	1.29	0.7	0.81	
		3+4	333	11.0	1.54	4.22	1.3	0.86	
		5	247	46.0	1.32	1.81	1.8	1.00	
	33-39	1+2	189	34.0	1.41	3.24	2.3	0.93	
	33 37	3+4	284	7.0	1.70	2.13	2.0	0.98	
		5	308	8.0	1.61	2.46	1.8	0.91	

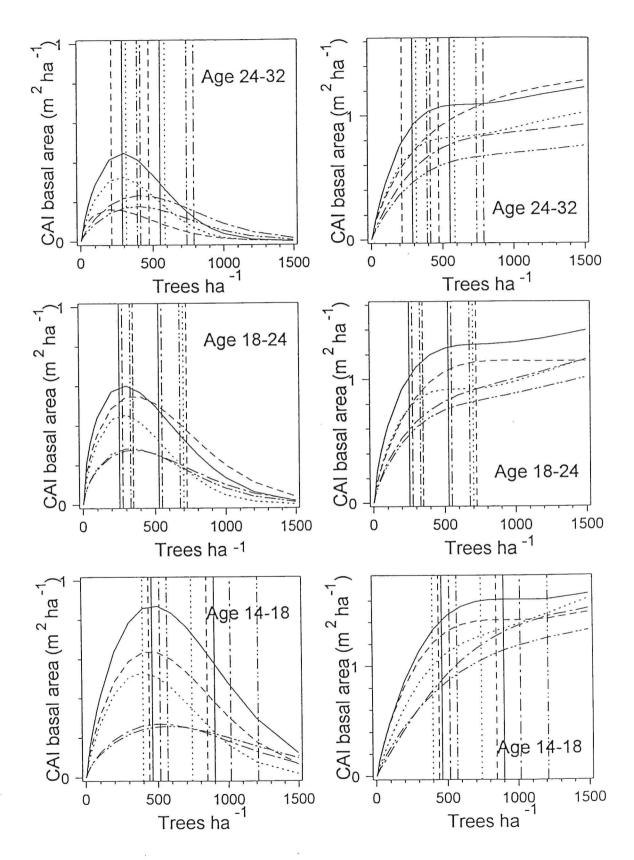


Figure 8. Position of the maximum and point of inflexion of release curves (Ir) for the five site qualities in the Free Growth Series. The positions are shown with respect to the Ir and the total increment (It) curves. Lines for site quality coded solid (Site 1), dashed (Site 2), dot (Site 3), dash and one dot (Site 4) and dash and two dots(Site 5) are similar for Ir maximum and Ir point of inflexion.

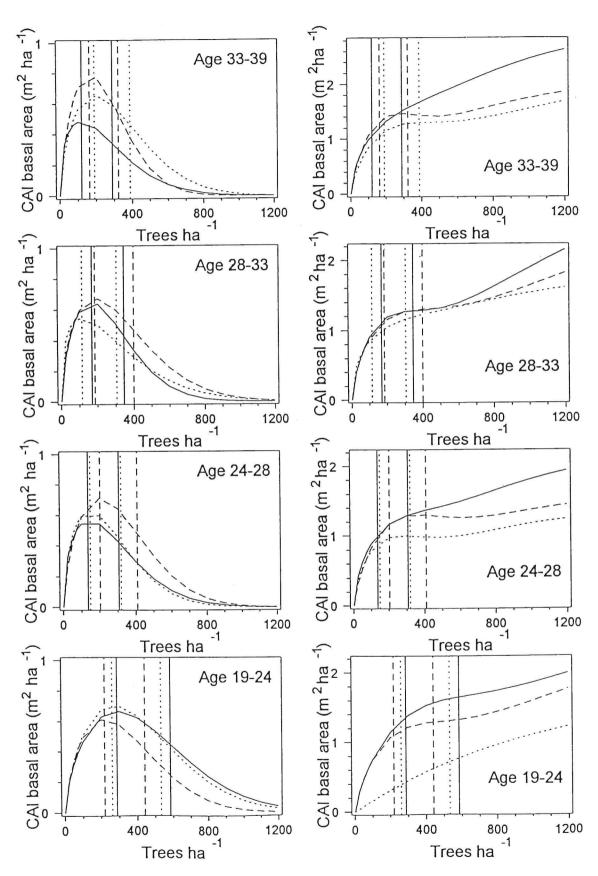


Figure 9. Position of the maximum and point of inflexion of release curves (Ir) for the three site groups in the Basal Area Control Series. The positions are shown with respect to both the Ir and the total increment (It) curves. Lines for site quality coded solid (Site 1+2), dashed (Site 3+4) and dot (Site 5) are similar for Ir maximum and Ir point of inflexion.

Age Variation - Stand density (Basal area, Table 2) for the critical points increased with age for both the high quality Late thinning Trial and for each site for the Free growth trial. The increase was lower in the Basal Control Trial.

Practical Application - Table 2 lists the stocking, basal area and basal area increment at these critical points for the P. pinaster data. Values enclosed in brackets for each critical point are the percentage of the increment at the critical point to that of increment of the control. For individual analyses the assumptions that Ir at point of inflexion equals the highest increment varied from 60 to 103 percent with a mean value for all locations, sites and age classes of 81 percent. Using this assumption in modelling to obtain full production with the minimum stocking could hence result in a 19 percent loss in volume. This may not be a worry in practice as obtaining the further 19 percent by increasing stocking will involve increased small sizes, at least part of which are not recoverable. Data in Table 2 suggests that the retention of planted stems of 1500 ha⁻¹ or more until a first thinning for particle board at stand ages 14-18 years will ensure maximum production in sizes acceptable to the product in question. Reduction to 500-600 s ha⁻¹ for the mid period 18-25 and then for age 25 onwards would quarantee maximum 400-500 production over the rotation.

The critical stocking at Ir maximum would optimise production of larger piece sizes per tree. Table data indicates that 400-500 s ha⁻¹ would generally achieve this at age 14-20 years with a loss of total volume of about 20 percent. Stockings of 250 to 350 and 150 to 250 s ha⁻¹ would optimise piece size for the 20-30 year and 30-40 year stand stages. Prior to considering such stockings in practice it is desirable to carefully analyse the price paid for logs or pieces of varying diameters. Locally there is a top diameter limit for price increases and no extra return is available for producing pieces above this limit. Optimum piece size is thus more often a matter of producing the maximum number of pieces to fit the diameter-price criteria. This does not necessarily correspond to the maximum response from thinning.

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