Department of Conservation and Environment

Water Level Maintenance Requirement Study for Lake Jandabup Western Australia

CR 240

June 1984

Environmental Consultants

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for Lake Jandabup, Western Australia

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for : The Department of Conservation & Environment
Western Australia

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WATER LEVEL MAINTENANCE REQUIREMENT STUDY FOR LAKE JANDABUP, WESTERN AUSTRALIA

Introduction

The purpose of this study was to specify the minimum late summer water requirement of Lake Jandabup to maintain it as a viable ecosystem, and to determine the maximum permissible drought groundwater allocation to pumping.

The need for the work has arisen because the confined and unconfined aquifers of the northern part of the Swan coastal plain are subject to pumping by the Metropolitan Water Authority for urban water supply, and by private landholders, mostly for irrigation of market gardens. The lakes of the Swan coastal plain are superficial expressions of the unconfined groundwater table, and could obviously dry up permanently if the water table level fell too far.

This brief study was undertaken to complement the urban water balance study currently in progress, and has focused on Lake Jandabup as a specific case of a problem common to most metropolitan wetlands north and south of the Swan estuary; that of the conflict in allocation of groundwater from the unconfined aquifer to wetlands versus human use.

The Study Team

This project was managed, and the ecological component of the work done by Dr. Robert B. Humphries of Natural Systems Research Pty Ltd.

The hydrological work was carried out by Dr. Lloyd R. Townley of the Centre for Water Research, University of Western Australia.

Methodology

It was proposed to :

- Reconstruct a water budget for the lake from 1968 onwards, the period for which lake water level data exist
- Compute level-volume, level-area and volume-area relationships for the lake
- Identify the short-term (annual) and long-term sources of variation in the water level record
- Evaluate, using literature sources and interview, the probable ecological consequences of (long-term) low water levels, and to develop recommendations for the minimum water levels necessary for maintenance of a desirable aquatic ecosystem.

Early examination of the available information and water level data revealed that the problem was too complex for the time and resources available, and that a full analysis of the hydrology of the lake depended in turn on a comprehensive analysis of the level data available for the unconfined aquifer. This would be most properly done by the Metropolitan Water Authority, and a suggested approach is outlined in Appendix A.

Some analysis of the hydrology of Lake Jandabup has already been done by Allen (1980), and by Anson (1983). These studies are discussed in detail in Appendices A and B respectively, but it should be understood that neither provide an adequate means of estimating the 'safe' amounts of groundwater abstraction which would ensure the continuation of a viable aquatic ecosystem in Lake Jandabup.

Furthermore, a 'safe' pumping limit is not a fixed figure - available quantities of groundwater will vary year-to-year, and seasonally. Unfortunately, the greatest demand for groundwater by both wetlands and for human use will coincide during times of drought.

This study therefore addresses the brief in the following ways :

- The questions relating to a water budget for the lake are discussed in Appendices A and B, and recommendations are made for improvement of existing analyses of the available data, as well as for specific tracer studies for Lake Jandabup.

- The computations of the level and volume-related curves for Lake Jandabup are not possible until the bathymetry of the rush-covered area of the lake is known. The calculations of ground water demand for Lake Jandabup are based on Allen's (1980) estimate of the lake volume. These are discussed in the following sections.

- Approaches to better identify the short-term (annual) and longer-term components of variation in the lake water level record are discussed in Appendices B and C.

- Some of the ecological considerations of lake water level variation are addressed in the following sections of this report.

Approximate Groundwater Demand by Lake Jandabup

Given the large difference between annual rainfall and annual pan evaporation near Perth, it is clear that if Lake Jandabup is managed so that there is always some area of open water, there will always be a net annual demand placed on the aquifer to satisfy the evapotranspiration needs of the lake. Even when the lake surface area reduces in summer, the surrounding reeds will continue to transpire at nearly the same rates as open water. A typical pan-to-lake coefficient for evaporation from shallow lakes would be 0.8.

Analysis of 15 years of data (1967 - 1981) obtained at the Perth Regional Office of the Bureau of Meteorology shows that the mean and standard deviation of annual rainfall are $m_r = 797$ mm and $\sigma_r = 155$ mm, while the corresponding statistics of annual Class A pan evaporation are $m_e = 1828$ mm and $\sigma_e = 160$ mm. The covariance σ_{er} between rainfall and evaporation is 1913 mm^2, hence the correlation coefficient is fairly small with a value of 0.07. Define the groundwater demand (the amount of water needed from groundwater to maintain water budget evaporation demand on the lake) to be

$$d = ce - r$$

where c is the pan-to-lake coefficient. Then the first and second moments of d are (i) the mean demand (Benjamin and Cornell, 1970, p.168) :

$$m_d = cm_e - m_r$$

and the variance :

$$\sigma^2{}_d = (c^2\sigma^2{}_e + \sigma^2{}_r - 2\ c\sigma_{er})$$

With c = 0.8, it follows that $m_d = 665$ mm and $\sigma_d = 193$ mm.

Given the statistics of d, and assuming that e and r are normally distributed, the calculated probabilities of exceedance of different demand values (Benjamin and Cornell, 1970, p.655) are shown in Table 1.

Table 1

Probabilities of exceedence for different annual groundwater demand values for Lake Jandabup, representing the demand for groundwater to maintain lake water balance.

Groundwater Demand (d) (mm)	Groundwater Volume** ($m^3 \times 10^6$)	p ($d \leq D$)	p ($d > D$)	Recurrence Interval (years)
665	2.66	50%	50%	2
795	3.18	75%	25%	4
912	3.65	90%	10%	10
983	3.93	95%	5%	20

* Based on 15 years (1967 - 1981) rainfall and evaporation data for Perth.

** Equal to demand d over the lake area of 4 km^2.

The above analysis could be improved in several ways :

(i) Although the available rainfall data for Wanneroo (009120) and Gnangara Forest (009119) do not differ statistically from those of Perth for the same years (One-way ANOVA, p>>0.05), longer data records for rainfall and evaporation for sites nearer the lakes should be used.

(ii) The evapotranspiration losses of water from the lake are poorly known. The pan-to-lake coefficient of 0.8 is approximate, and further investigation is required.

The groundwater demands necessary to maintain lake levels should be compared with the 10 - 12 x 10^6 m^3 pumped by the MWA from the unconfined aquifer, and the 13.3 x 10^6 m^3 allocated for private abstraction from the unconfined aquifer. The designed annual output of the MWA's Wanneroo Scheme is 12.2 x 10^6 m^3; this rate was only achieved in 1979 - 1980. Lower rates of abstraction have been typical since the inception of the scheme in 1973 (Metropolitan Water Authority 1984). The locations of the MWA bores in the Wanneroo Groundwater Area and their relationship to the major lakes are shown in Figure 1.

The licensed pumping of groundwater from the unconfined aquifer in the vicinity of Lake Jandabup is more of a cause for concern. The private bores are apparently not metered, and therefore usage can only be estimated. The current allocation to private users within the boundary marked in Figure 2 is about 13.3 x 10^6 m^3, but this allocation is not fully utilized.

Although neither the MWA nor private users are taking their full allocation of groundwater, there has been a declining trend in the levels of Lakes Jandabup and Gnangara since 1976 (Figure 3). Further, there has been a decline of about one metre in the level of the unconfined aquifer surface in the vicinity of Lake Jandabup from April/May 1976 to April/May 1983 (Figure 4).

Factors influencing the fluctuations of the water table in the region include (Metropolitan Water Authority 1984) :

- variations in annual rainfall, and hence groundwater recharge. (The 10 year moving average rainfall has been below the long-term average since the mid-1960's, see Figure 5)
- changes in forest evapotranspiration rates. (The water demand of the Gnangara pine plantation has probably increased steadily during the 1970's, as the trees mature)
- groundwater abstraction for public water supply by the MWA
- increasing private groundwater abstraction for irrigated agriculture
- changing (increasing?) evapotranspiration rates from wetlands

The MWA (Metropolitan Water Authority 1984) concludes that the decline in lake water levels correlates closely with monthly variations in evaporation and rainfall, and not with MWA pumping. The cause of the decline in aquifer levels ie attributed to below average rainfall. We feel that the analyses of data completed to date (e.g. Allen 1981, Anson 1983, MWA 1984, see Appendices) do not unequivocally demonstrate that groundwater abstraction has no effect on lake levels, and that further data analysis using computer modeling (Appendix C) is necessary before firm conclusions can be drawn.

The computer modeling studies would interpret the monthly multi-level piezometric data available from bores within at least a 5 km radius of Lake Jandabup. A greater area of interest would be better still, because the dynamics of neighbouring wetlands could also be investigated. Analyses of lake level variation alone are insufficient - the behaviour of the aquifer must be known to adequately account for the effects of groundwater abstraction.

The short- and long-term sources of water level variation of both wetlands and the unconfined aquifer were investigated by the MWA and GSWA before the Wanneroo pumping scheme began (Pollett 1981). This present study cannot hope to improve upon that work.

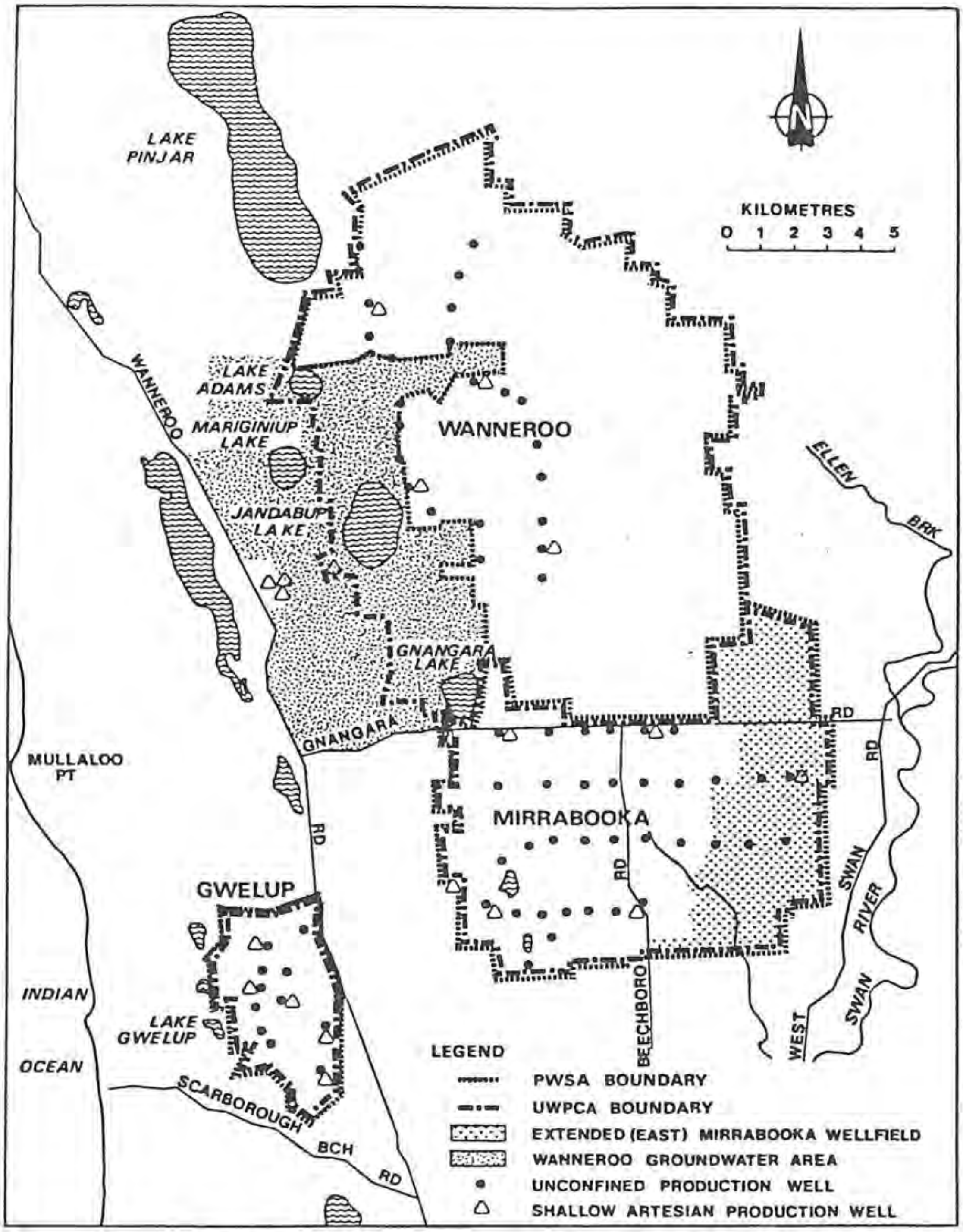

FIG. I - LOCATION OF GROUNDWATER SCHEMES IN THE NORTHERN PERTH AREA

Source MWA 1984

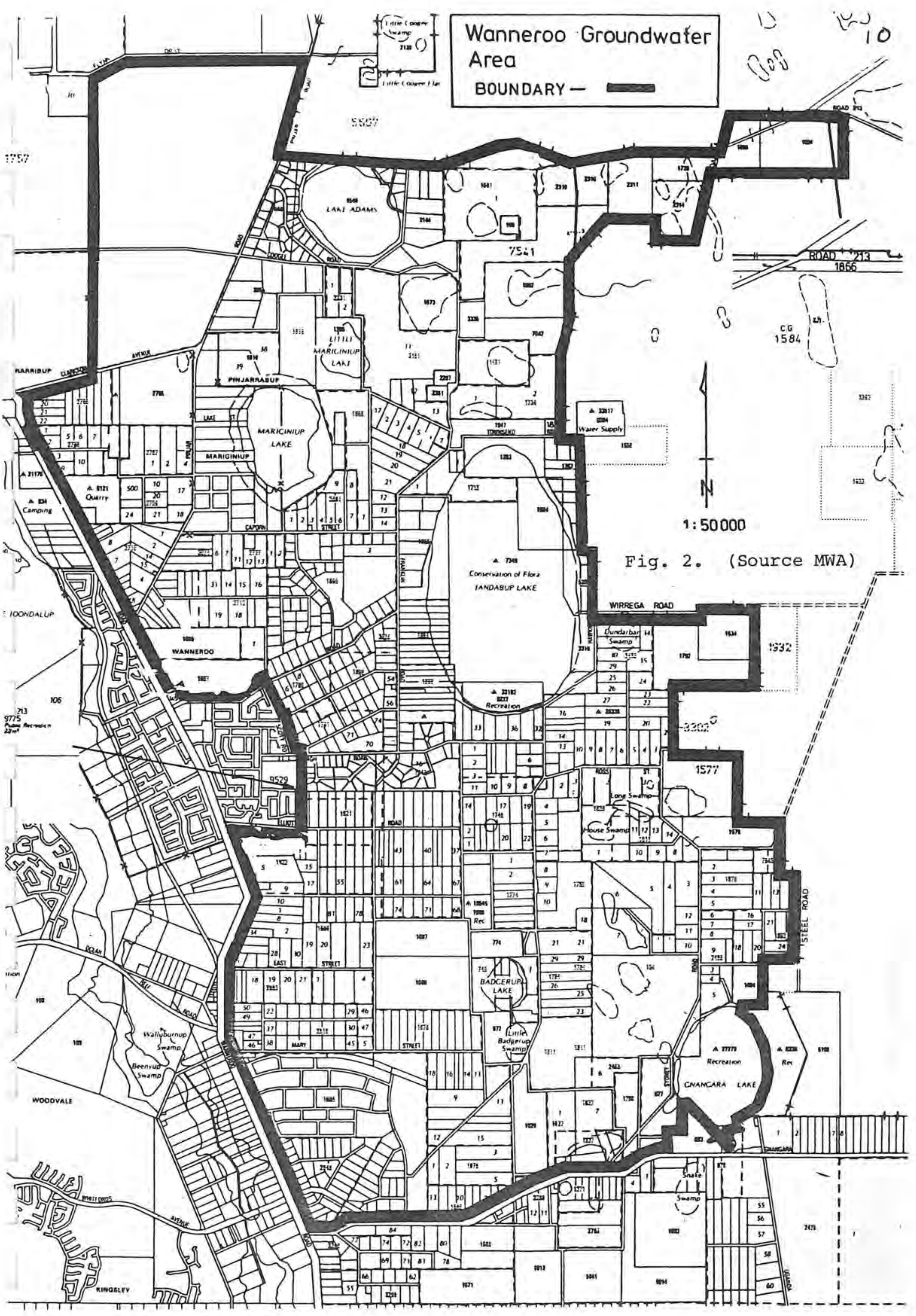

Fig. 2. (Source MWA)

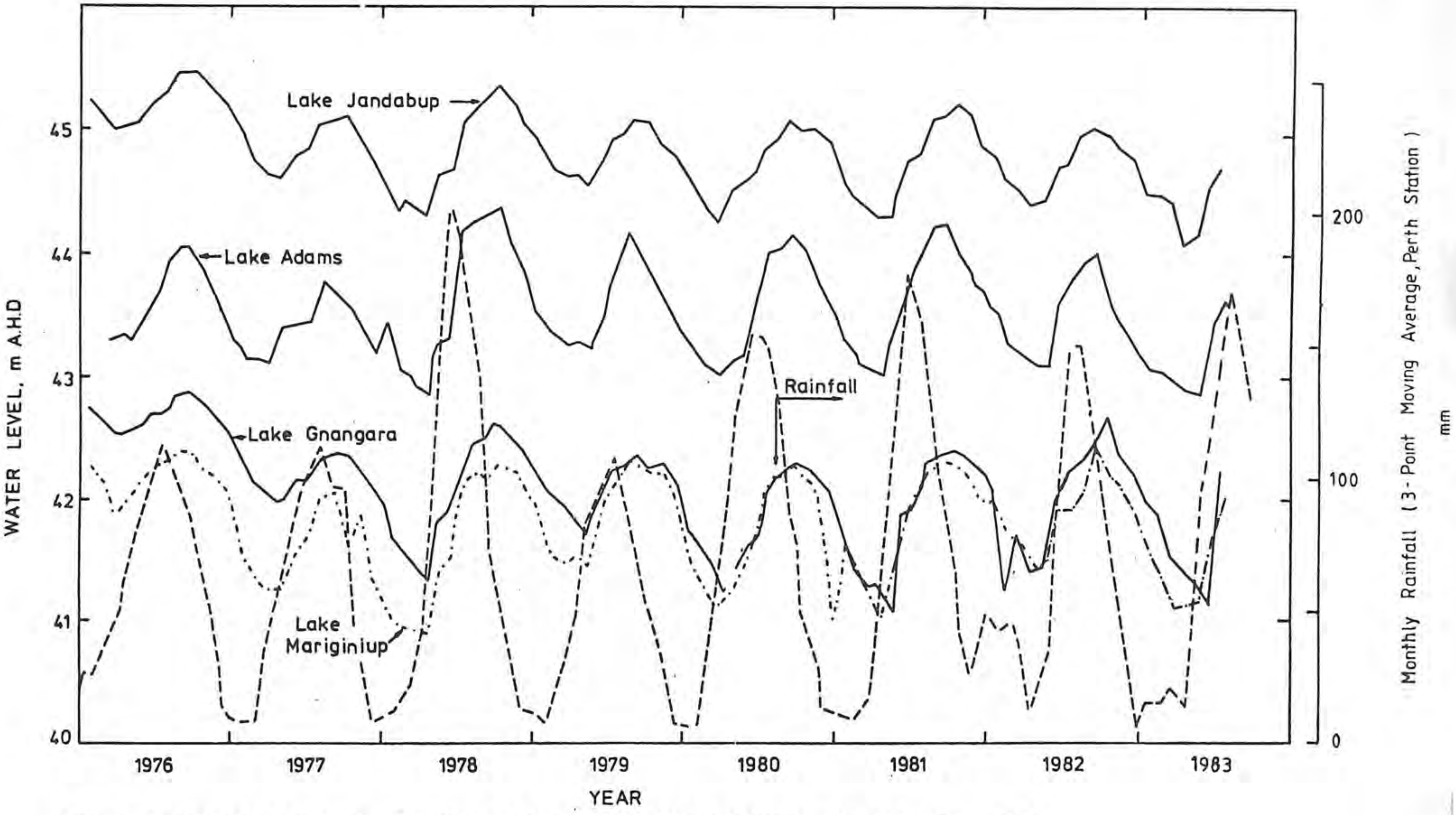

FIG. 3 HYDROGRAPH OF MAJOR LAKES IN WANNEROO WETLAND AREA

Source: MWA 1984

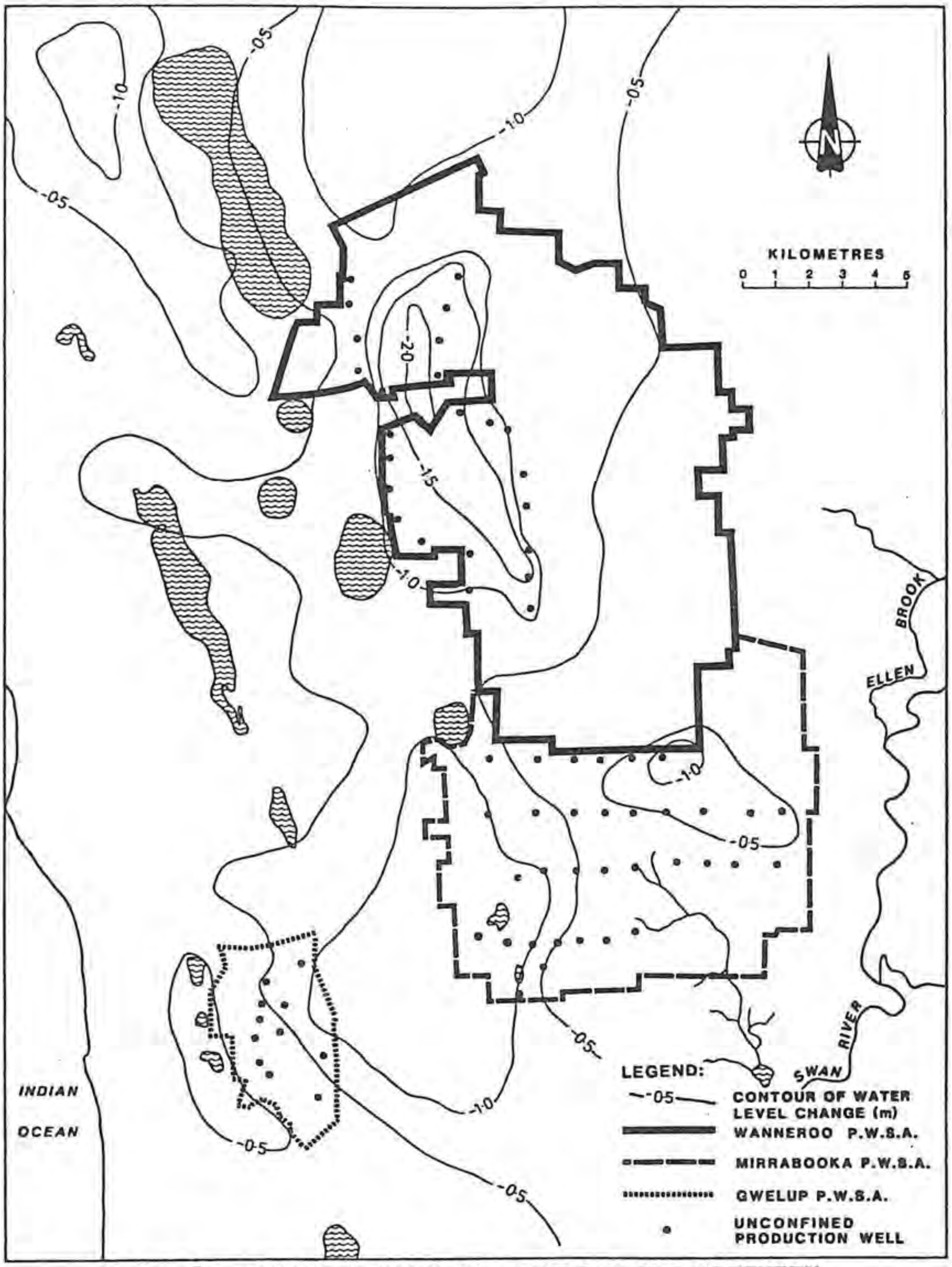

FIG. 4 - CHANGES IN WATER LEVELS IN THE NORTHERN PERTH AREA : APRIL/MAY 1976 - APRIL/MAY 1983

Source MWA 1984

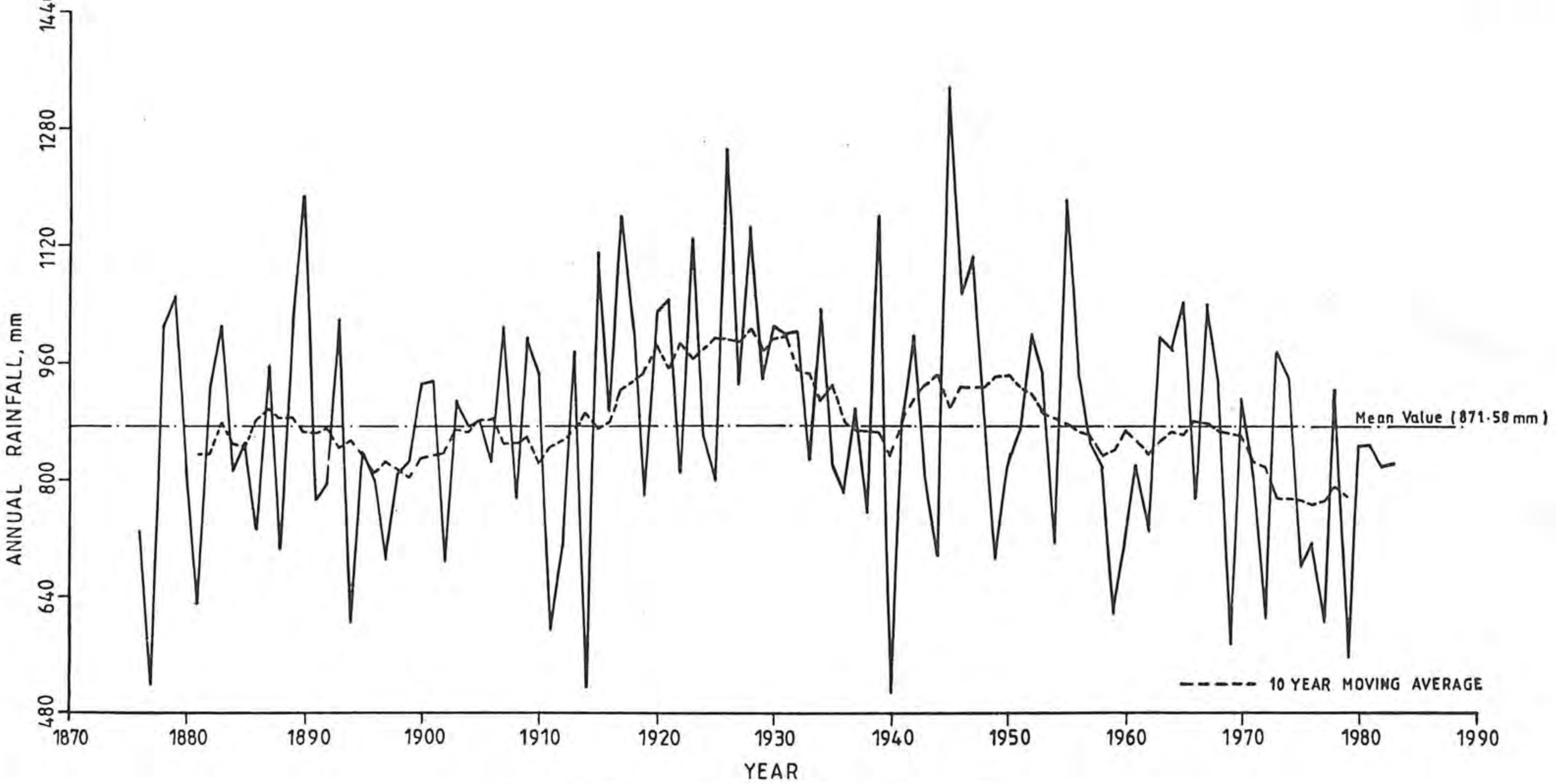

FIG. 5 PERTH ANNUAL RAINFALL
(1876 to 1983)

Source: MWA 1984

The Ecological Significance of Lake Water Level Variation

The water levels of all coastal plain wetlands vary seasonally, and the biota using these habitats is adapted to the seasonal changes in water availability. Indeed, water level variation is an essential ecological requirement of many species.

The level of Lake Jandabup has been observed to vary between 43.58 to 46.78 m AHD, with a mean of 45.29 m AHD, although the level record, beginning in September 1954, is incomplete. Table 2 contains a list of the annual minimum levels recorded since 1954.

Approximately monthly measurements have been taken since 1969; the annual minimum level is usually reached in April, and the maximum level is attained in late September or early October.

Jandabup is a notable habitat for several species of plants and animals.

- Three species of bladderworts (Utricularia spp.) grow in the south-east corner of the lake, where a fresher-water inflow occurs. Two of the species are unusual in this locality (Lantzke 1983). Bladderworts require seasonal inundation for the aquatic phase of their life cycle, and then flower as the sediments beneath them dry in summer.

- The spike-rushes Baumea articulata and B. juncea dominate the vegetated areas of the lake, rather than the aggressive bullrush, Typha domingensis, which only occurs in small patches. Typha is known to replace Baumea spp. if wetlands are greatly disturbed, but the ecological consequences of such replacements are not understood. The open water area of Lake Jandabup is about 120 ha (34%) of the total lake area at maximum water level. Baumea articulata occupies areas flooded to 50 - 60 cm, and B. juncea is confined to areas flooded to about 30 cm (N. Marchant pers. comm.).

  A reduction of the open water area by rush encroachment would occur if either the period of maximum water levels, or the maximum water levels of the lake were significantly reduced.

- Jandabup is a significant habitat for rush-dwelling and wading birds on the coastal plain. Thirty-six species of birds have been recorded, and at least four species are known to breed there. (Pearson, M.S. 1983). The most significant species are four transequatorial migrant waders, which exploit the exposed sand- and mudflats during times of falling water levels, particularly in late summer and autumn, and little bitterns, which have recently been recorded breeding in the Baumea rush beds (Jaensch 1984). The diverse bird species supported by the lake utilize all three major habitats - the rush beds, open water and open flats, and the number of species present would probably decline if the diversity of habitats were reduced.

  The coastal plain wetlands are important late summer/autumn habitats and drought refuges for water birds, who are forced to leave inland wetlands as they dry out. The longer persistence of water, and the seasonal changes in water level are both components of the attractiveness of these habitats to waterfowl.

- The coastal wetlands also support a diverse biota of aquatic invertebrates, algae and higher water plants (Ayre et al 1977, Western Australian Museum 1978, Carbon 1976, Loneragan et al 1984).

Table 2

Minimum water levels recorded for Lake Jandabup (Source : Metropolitan Water Authority).

Month of minimum recording		Level (m AHD)	No. of observations per year	Comments
Sept.	1954	45.02	1	
April	1955	43.87	1	
Sept.	1956	45.60	1	
April	1961	43.78	2	
May	1962	43.48	2	
Oct.	1963	45.67	1	
Oct.	1964	46.10	1	
Aug.	1968	46.50	8	All observations after August
April	1969	46.13	16	
June	1970	46.06	6	All observations after August
May	1971	45.48	11	
May	1972	45.18	11	
May	1973	44.76	11	
May	1974	44.76	11	No April observation
May	1975	44.99	11	
April	1976	44.96	11	
May	1977	44.57	12	
April	1978	44.26	12	
May	1979	44.52	12	
March	1980	44.18	12	
April	1981	44.25	12	
April	1982	44.34	12	
April	1983	44.06	12	
April	1984	44.24	4	No observations after April
MEAN		44.75		

Note: The mean minimum level is calculated on 14 years' data, 1969 - 1983, excluding 1970.

Conclusions and Recommendations

(i) The lack of correlation of MWA pumping and Lake Jandabup level variation is insufficient evidence on which to assume that groundwater abstraction has no effect on lake levels. It is recommended that the Metropolitan Water Authority thoroughly analyse the dynamics of the unconfined aquifer in the region of Lake Jandabup and report their findings before any new, large groundwater allocations are made. Suggestions for the appropriate analyses are contained in Appendix C.

(ii) Better data on the private use of groundwater are needed, in both time and space.

(iii) The population dynamics of wetland plants and animals is closely tied to the seasonal cycle of water level variation. Long-term decreases in maximum water levels will encourage invasion of open water areas by rushes, and result in the transition of lakes into fens, like Herdsman Lake.

The phenology and ecology of the major rush species is poorly understood. Carefully designed studies of rush biology relevant to management of wetlands should be supported, as should aerial photographic and ground level monitoring of wetland vegetation change.

(iv) The management of coastal wetlands should be considered on an individual wetland-by-wetland basis, as well as on a regional (aquifer-wide) context, because the wetlands are linked via the unconfined aquifer. Greater synthesis of existing information, and integration of new and existing hydrological and biological studies would facilitate this.

(v) More data on depth-to-water table and lake evapotranspiration rates are required for better modelling of the unconfined aquifer and the lakes of the coastal plain.

(vi) Knowledge of the bathymetry of Lake Jandabup and other coastal plain wetlands is a necessary component of the information required for wetland management. The Metropolitan Water Authority should be encouraged to obtain bathymetric information for all significant wetlands likely to be affected by pumping.

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APPENDIX A

Water Balance Computations

A.1 Introduction

The notion of a 'water balance' implies that a boundary can be defined around a hydrological system in such a way that all fluxes through the boundary and storage changes within the boundary can be accurately assessed. Lake Jandabup is a surface expression of the Gnangara Mound flow system, and its level is maintained by dynamic interaction between the atmosphere and the groundwater resource (Allen, 1980). It follows that we can meaningfully consider at least two types of water balances : one for the lake alone and another for a larger system containing both the lake and the aquifer.

Water balance computations may also be performed on different time scales. In general, the larger the time interval, the more we can neglect (or average out) the details of the physical processes taking place. On the other hand, if a water balance model is to be used predictively to assess the effects of changing physical processes, those processes need to be realistically included in the model, and the chosen time interval needs to be appropriate for those processes.

A.2 Water balance of Lake Jandabup alone

In this case we effectively define a control volume in the shape of a horizontal disk, with its bottom boundary just below the lake sediments, its top boundary just above the highest water level, and its side boundaries at the most extreme shoreline. A general water balance equation for the volume of water stored in the lake takes the form

$$\frac{\Delta v}{\Delta t} = \sum \text{inflows} - \sum \text{outflows} \qquad \text{(A1)}$$

where Δv is the increase in lake volume during the time interval Δt and flows are expressed in units of $[L^3/T]$. Inflows include rainfall on the water surface (and on nearby areas which contribute surface runoff) and groundwater inflows, while outflows include evaporation, transpiration by reeds and groundwater outflow. All the inflows and outflows are difficult to measure directly and independently. Water balance calculations of this type are extremely sensitive to errors, especially since the small change in storage volume is calculated as a difference of much larger volumes (Winter, 1981).

Allen (1980) computes the annual water balance for Lake Jandabup between April 1977 and March 1978. Undoubtedly his balance is of the right order of magnitude, although a possible weakness in his work is that the average groundwater inflow and outflow throughout the year are based on observed piezometric heads on only two occasions during the year (October 1977 and March 1978). Allen's method of interpolating equipotentials between sparse data and then estimating flow lines is prone to error; this may explain his use of only two months' data when twelve months' data were available.

Allen (1980) correctly identifies the interaction of the lake and the unconfined aquifer as involving the convergence of flow lines, both laterally and vertically, towards the lake (see Figure A.1).

There are two reasons, however, why Allen's inflow and outflow estimates may be inaccurate. Firstly, his regions of flow "involving" the lake (his Figure 7) are probably too thick, since a dividing flow line should intersect the lake bottom at a single point. Secondly, neglecting three-dimensional effects (i.e. assuming a rectangular window of dimensions b and L) will tend to exaggerate the results. Both of these factors would lead to overestimates of flows, however Allen's net groundwater inflow may still be roughly correct.

Seasonal variations may be a cause of greater error, because the average wet season is shorter than half the year. In a dynamic system, we would expect the position of dividing flow lines to fluctuate seasonally. In particular, their points of contact with the lake would move, defining a line on the lake bottom, separating the regions receiving water from or recharging water to the aquifer. It seems possible that at some time of the year, Lake Jandabup may actually be a net source of recharge for the aquifer (see Figure A.2).

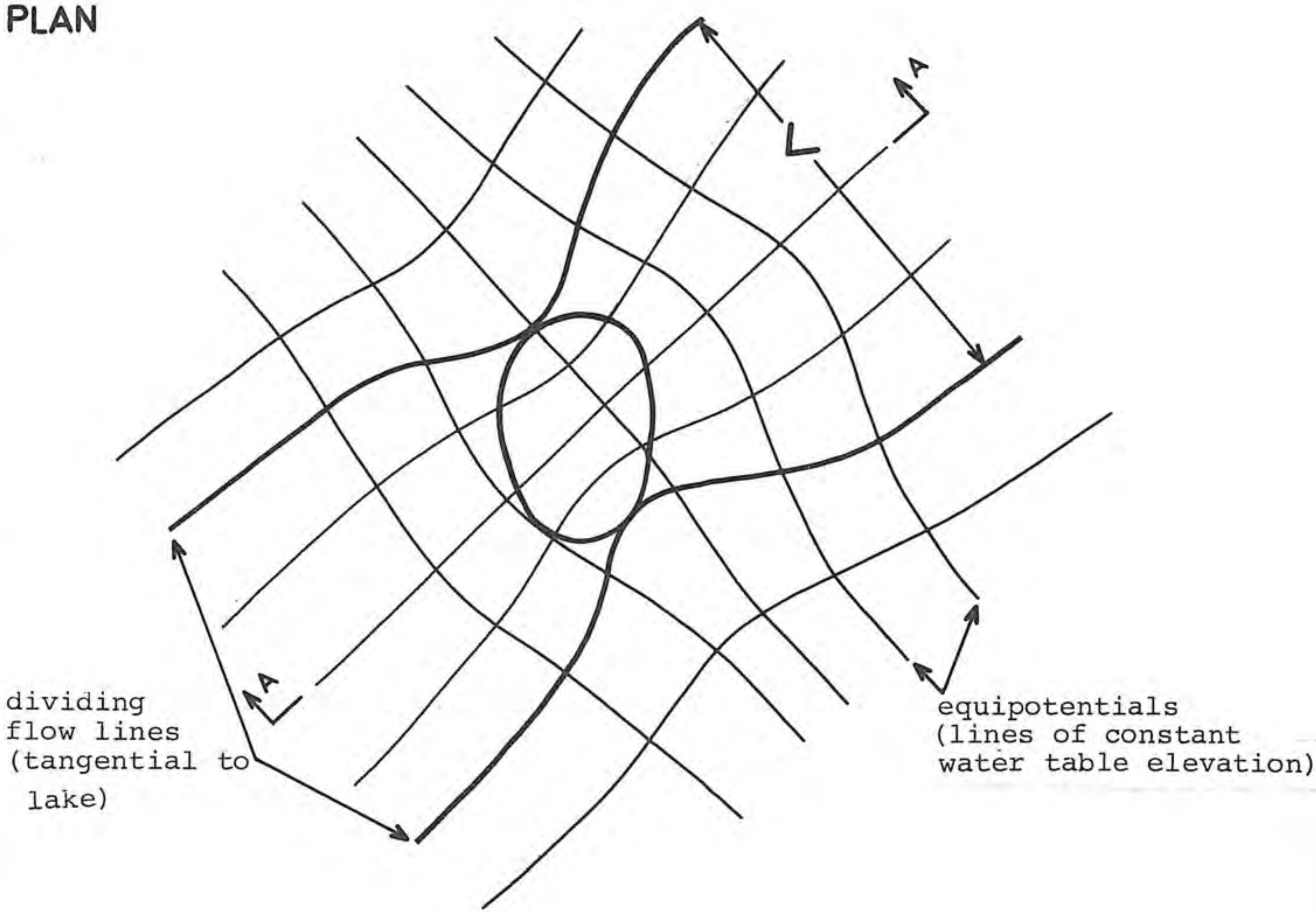
The effect of partially-penetrating rivers and streams (which are narrow compared with the scale of horizontal variations of piezometric heads) is discussed by numerous authors (see qualitative discussion by Bear, 1979, pp. 51-53, 163; references quoted by Townley and Wilson, 1979, pp. 65-77). However the hydraulics of interaction between very shallow lakes and unconfined aquifer does not seem to be well documented in the literature. This topic seems worthy of fairly detailed investigation by the MWA. A reasonable approach would be to use two-dimensional finite element models in cross-section, in order to develop an understanding of the variation with depth of piezometric heads beneath the lake; models of this type would account for the effects of the low permeability diatomite layer, which would require steep gradients across its thickness in order to pass large flows. It may then be possible to develop "effective leakage coefficients" for lake elements in a horizontal finite element model based on the hydraulic approach (Townley and Wilson, 1979, p.21).

Given that monthly piezometric heads at nested bores are available since 1977 at a number of sites, it seems likely that by calibrating a lake-aquifer interaction model, a monthly water budget could be established. Another possibility, however, would be to use data other than heads to achieve this goal. We note at the outset that fundamentally the three most difficult quantities to estimate are evapotranspiration and groundwater inflow and outflow. If concentrations of chloride and another tracer (e.g. tritium) were measured on a monthly basis in rainfall, in the lake and at bores upstream and downstream of the lake, then simultaneous solution of Allen's (1980) Equations (1) and (3) and Turner et al's (1983) Equation (2) would lead to monthly estimates of all three quantities.

A.3 Water balance of Lake Jandabup and the surrounding aquifer

In order to assess the effects of pumping on Lake Jandabup, a water balance is really needed for a larger region enclosing both the lake and the area to be pumped. The difficulty here is that whereas a lake surface can reasonably be assumed to be horizontal (hence a single level defines the volume stored), the water table elevation in an unconfined aquifer varies considerably in space. A water balance model for the aquifer must therefore satisfy the equations of flow in porous media, which account for the porosity and permeability of the aquifer material. There are, however, several possibilities within this framework.

PLAN



ELEVATION

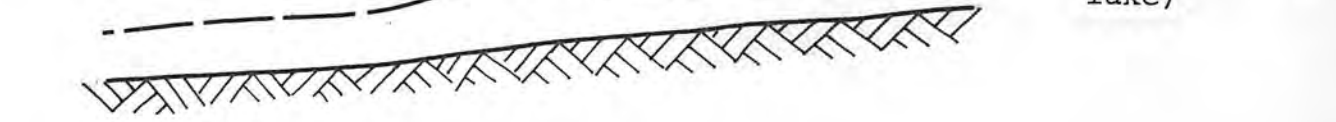
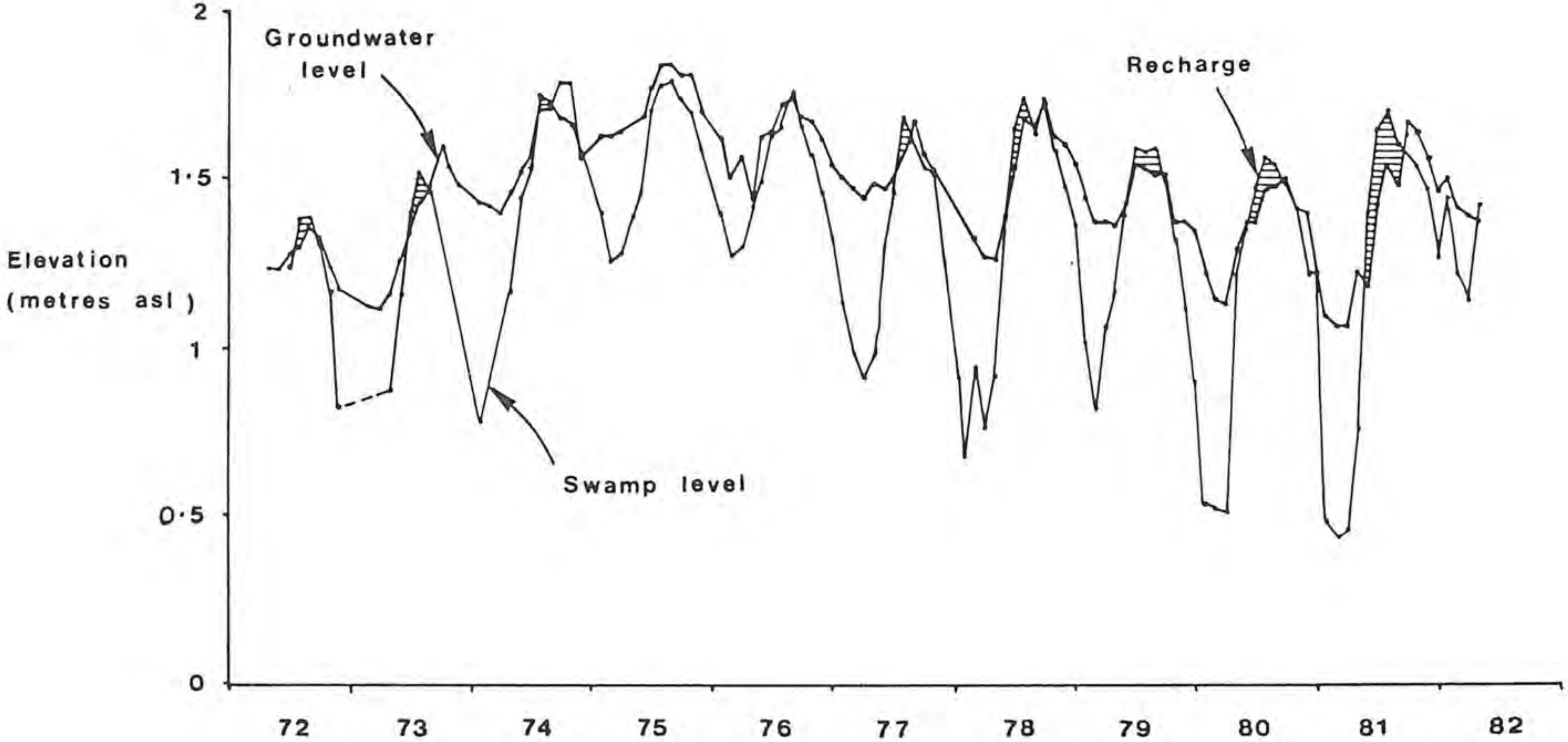


Figure A1 Schematic diagram showing flow convergence towards a typical lake.



Groundwater and surface water levels at Star Swamp (735101).

Appendix Figure A2 - from Loneragan et al 1984

The three-dimensional equations of porous media flow can be simplified to one or two dimensions in some circumstances. One-dimensional models are unlikely to be useful for Lake Jandabup, although they may be useful as a learning tool. Two-dimensional depth-averaged models will probably be useful, but as mentioned above, they will require some preliminary analysis to allow the vertical flow effects beneath the lake to be properly simulated; (for example see Winter 1983). An unresolved and difficult problem is the definition of boundaries for the model, where appropriate boundary conditions are known.

A.4 Recommendations on water balance modelling

The M WA should be encouraged to consider the following studies :

(i) The use of chloride and tritium measurements (or some other suitable tracer) to define the monthly water balance of the lake itself.

(ii) The study of lake-aquifer interaction using cross-sectional models to develop effective leakage coefficients for lake elements in two dimensional aquifer models.

(iii) The use of two-dimensional aquifer models to assess the effects of the spatial and temporal distribution of pumping on the regional flow system and more specifically on water levels in Lake Jandabup.

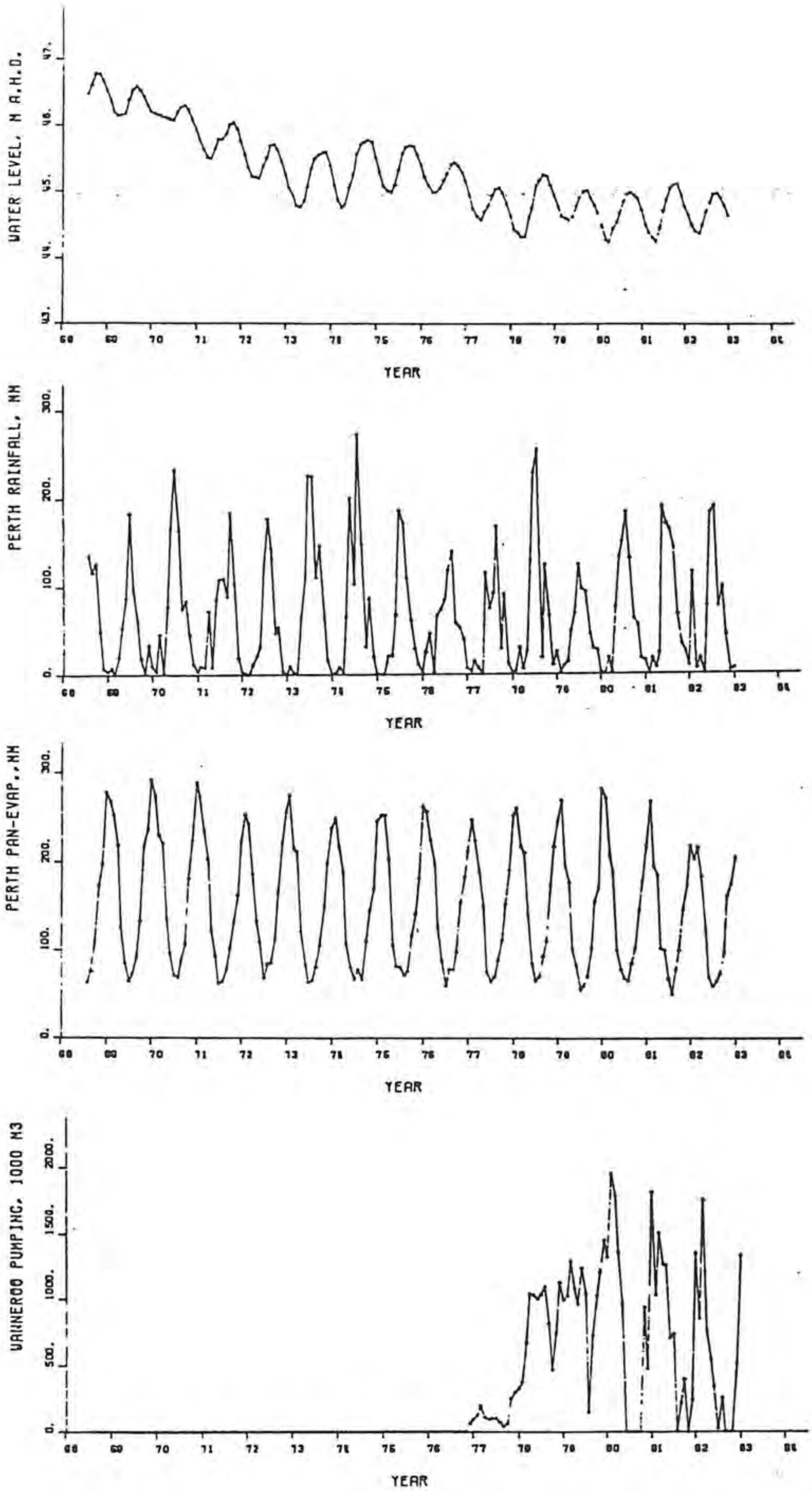
APPENDIX B

The Use of Regression Models

A major difficulty with statistical models is that correlation between variables does not imply causality, i.e. that statistical relationships do not necessarily result in good predictive tools. A corollary is that if a statistical relationship is not found, there is insufficient evidence to say that a deterministic relationship does not exist.

The work by Anson (1983) is a reasonable attempt to use regression analysis to model the water balance of Lake Jandabup. The component data used for his analysis are plotted in Figure B.1. The structure of the chosen regression model is very similar in form to a deterministic water balance model of the form of (A.1). If groundwater inflow and outflow are approximately linear combinations of rainfall and evaporation in the present and preceding one or two months, then Anson's regression coefficients effectively attempt to model the groundwater regime as well.

The reason why pumping is not statistically significant is not immediately clear, although the lack of periodicity compared with rainfall and evapotranspiration may play a role. Often it is useful to allow regression coefficients to vary seasonally when the dependent and independent variables have obvious seasonal trends. A number of extensions to Anson's method are possible, however at this stage the regression model does not prove that pumping has no effect; it simply shows that levels can be reasonably represented by only considering rainfall and evaporation.



Lake Jandabup: Major Input Variables
Source: Anson 1983
Appendix Figure B1

APPENDIX C

Groundwater Hydraulic Management Models

C.1 Introduction

According to Bear (1979, p.10, see also Gorelick, 1983) the management of a groundwater resource system aims at achieving certain goals through a set of decisions or policies related to the development or operation of the system. Typically the same goals may be achieved by different policies, thus management requires the selection of policies which are in some sense the 'best'.

With reference to the management of the groundwater resource in the vicinity of Lake Jandabup, different policies could be defined by the values of 'decision variables' representing private pumping rates in a number of zones. An optimal management strategy might be chosen to maximise either the total abstraction over all zones, or some weighted total which takes into account economic or even intangible benefits; this scalar function of the decision variables is known as an 'objective function'. A management 'constraint' would be that the surface water level of Lake Jandabup should always be greater than some specified minimum level for each month of the year.

A large body of literature exists on the analysis of 'optimisation' problems which have the general form:

$$\max_{\underline{u}} \quad J(\underline{u}) \qquad \text{(C1)}$$

subject to :

$$f_i(\underline{u}) = 0 \qquad (i = 1,...,m) \qquad \text{(C2)}$$

$$g_j(\underline{u}) \geqslant 0 \qquad (j = 1,...,n) \qquad \text{(C3)}$$

where $\underline{u}$ is a vector of decisions variables (e.g. pumping allocations), $J(\underline{u})$ is the objective function (i.e. the criterion on which the optimal solution is to be based), $f_i(\underline{u})$ is an equality constraint (e.g. the requirement that the water balance must be preserved), and $g_i(\underline{u})$ is an inequality constraint (e.g. the requirement of water level maintenance in Lake Jandabup). The optimal solution can be obtained either by linear programming, when the objective function and constraints are linear functions of $\underline{u}$, or by nonlinear programming, when any of these functions are nonlinear in $\underline{u}$. Standard references describe these techniques (e.g. Simmons, 1972; Luenberger, 1973; Bradley et al., 1977).

In order to demonstrate the applicability of these methods to the Perth region, a series of examples of increasing complexity will be briefly discussed.

C.2 Linear programming for wellfield design

Consider a cluster of m fully-penetrating wells (screened over the full aquifer thickness) in an infinite confined aquifer. Let s_{ij} be the drawdown at the i'th well due to pumping at the j'th well at rate Q_j. Then, assuming homogeneous and isotropic aquifer properties,

$$s_{ij} = \frac{Q_j}{2\pi T} \; W \left(\frac{r_{ij}^2 \, S}{4Tt} \right) \tag{C4}$$

where S and T are the aquifer storage coefficient and transmissivity, respectively; t is the time since pumping commenced; r_{ij} is the separation between the two wells; and W is the well function (see, e.g. Bear, 1979, p.321). Because the governing partial differential equations are linear in this case (e.g. Bear, 1979, [8-54]), the principal superposition is valid and (C4) can be used even in the presence of an ambient flow field.

A typical problem would be the design of a wellfield to maximise output, subject to draw down constraints (see, e.g., Bear, 1979, p.498; Merrick and Drury, 1983). We seek to maximise

$$J\,(Q_1, \ldots, Q_m) = \sum_{i=1}^{m} Q_i \tag{C5}$$

subject to the constraints that drawdown at each well should not exceed specified limits, i.e.

$$s_i = \sum_{j=1}^{m} s_{ij} = \sum_{j=1}^{m} a_{ij} \, Q_j \leqslant s_{i,\,max} \quad (i = 1, \ldots, m) \tag{C6}$$

$$\text{where} \quad a_{ij} = s_{ij}/Q_{ij} \tag{C7}$$

and also subject to the positivity constraints

$$Q_i \geqslant 0 \quad (i = 1, \ldots, m) \tag{C8}$$

Since (C5) and (C6) are linear in the Q_i, this problem is easily solved by linear programming. Different optimal pumping rates would be obtained by applying the drawdown constraints at different times after the start of pumping. Furthermore, the results would also be affected by seasonal variations in the ambient flow field, which would probably alter the allowable drawdowns.

In an unconfined aquifer, superposition is not valid unless the governing equations can be justifiably linearised. The approach should be modified to account for non-fully-penetrating wells, and the maximum permissible drawdown would depend on the depth to screening, as well as on economic considerations. The effects of anisotropy (when horizontal and vertical permeabilities are unequal) should also be taken into account (see Pollett, 1981). Nevertheless, a very similar approach can be adopted, and standard linear programming packages, available in the NAG or IMSL libraries on most computer facilities, can be used.

C.3 Linear programming using numerical aquifer models

Consider a simple conceptualisation of the Jandabup situation, in which a one-dimensional finite difference model is used to simulate aquifer flow (see Figure C.1). (As mentioned earlier, a one-dimensional representation is probably inadequate for assessment of Lake Jandabup's flow regime but is sufficient here for purposes of illustration).

Let the region be divided into m zones (of length d_i) for which different rates of extraction (q_i per unit width per unit length) are to be allowed. Also suppose that piezometric heads are calculated at several nodes beneath the lake, and that leakage occurs upwards or downwards depending on the difference between aquifer head and the horizontal lake water level. The management problem can then be written

$$\text{Max } J(\underline{q}) = \sum_{i=1}^{m} q_i d_i \qquad \text{(C9)}$$

subject to

$$h_{L(\underline{q})} \geqslant h_{min} \qquad \text{(C10)}$$

where h_L is the modelled piezometric head at a node beneath the centre of the lake and h_{min} is large enough to allow sufficient upward leakage to satisfy evaporation demand and maintain the lake water level. (A more complete definition of the problem would place constraints on heads at all nodes within the lake).

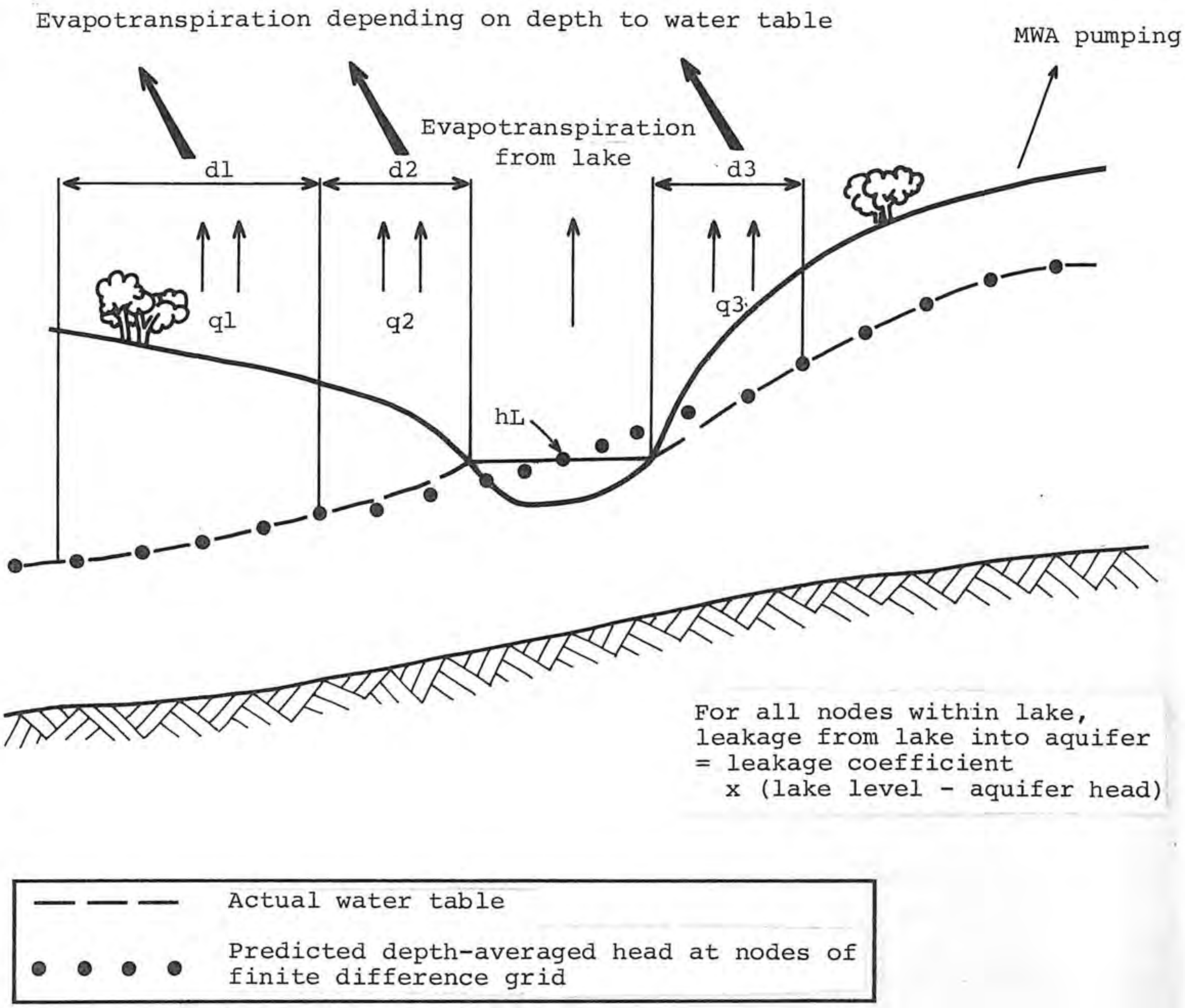
The difference between this problem and that of wellfield design is that simple analytical expressions are not available for relating the effect of each decision variable q_i to the constrained head h_L. Furthermore, since the aquifer is unconfined, the finite difference model is nonlinear in heads, and an iterative procedure is needed to linearise the constraint. It becomes necessary to solve a series of linear programming problems, each of which uses the best previous estimate of optimal $\underline{q}$ to solve for a new estimate. If $\underline{q}^*$ represents the best previous estimate, then the constraint (C10) can be linearised to the form

$$a_1q_1 + a_2q_2 + a_3q_3 \geqslant h_{min} - [a_1q_1^* + a_2q_2^* + a_3q_3^*] \qquad \text{(C11)}$$

where

$$a_i = \left. \frac{\delta h_L}{\delta q_i} \right|_{q = q^*} \qquad \text{(C12)}$$

The sensitivities a_i can be determined numerically each time a new optimum is found.



FigureCl One dimensional idealisation of flow system near Lake Jandabup.

C.4 Nonlinear programming with a model supplying constraints

A further level of sophistication is to simultaneously include the groundwater model as a set of constraints in a nonlinear optimisation problem (see, e.g., Bear, 1979, p.500). This is in contrast to the procedures described above, where firstly the model is used to determine sensitivities and only then are the sensitivities utilised in the optimisation problem.

A similar problem to that defined above would be seeking to minimise

$$J(\underline{q}) = -\sum_{i=1}^{m} q_i + W[h_L(\underline{q}) - h_{min}]^2 \qquad (C13)$$

where $h_L(\underline{q})$ is the modelled head at a lake for a given distribution of withdrawals, h_{min} is the desired piezometric head beneath the lake, and W is a (large) weighting factor. Incorporating the water level constraint in this way is known as a 'penalty function' formulation. If we adopt a steady state finite difference or finite element model of the form

$$\underline{K}\,\underline{h} = \underline{f} \qquad (C14)$$

(see Wilson et.al., 1979), then the constraining influence of the model can be built into the management problem by defining an equivalent objective function

$$J(\underline{q}, \underline{h}) = -\sum_{i=1}^{m} q_i + W[h_L(q) - h_{min}]^2 + \underline{\lambda}'(\underline{K}\,\underline{h} - \underline{f}) \qquad (C15)$$

In this case, $\underline{\lambda}$ is an adjoint vector (i.e. a vector of Lagrange multipliers), and $J(\underline{q})$can be minimised by standard nonlinear programming techniques (see, e.g., Townley, 1983).

C.5 Recommendations on management modelling

A prerequisite for optimal management planning is the existence of accurate water balance models of the type suggested in Appendix A. Once these are available, the MWA should develop linear and nonlinear programming techniques to optimise total use of the aquifer subject to environmental, water quality and economic constraints.