UNDERSTANDING GROUNDWATER MODELLING – CONCEPTUAL, ANALYTICAL and NUMERICAL

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Outline of Talk

- What is a Model?
- Why Model at All?
- Modelling Process
- Conceptual Model
- Analytical Modelling
- Numerical Modelling
- Modelling Guidelines

(Some examples along the way)

What is a Model?

- Physical or mathematical substitute
  - *e.g. toy truck; formula; software*
- Replica of a natural system
  - *but only the essential processes*
- An indispensable tool
  - *objective; quantitative; experimental*
- Computer models are now regulatory tools
  - *vital for management, protection, remediation*

Why Model at All?

- Framework for data synthesis
  - *"More data?"*
- Better understanding of complex interactions
  - *"What happened?"*
- Prediction scenarios
  - *"What if?"*
- Optimal management
  - *"What's best?"*
- Communication / Visualisation
  - *"Show me"*

Modelling Process

- Understand.
- Conceptualise.
- Select and apply the correct model …

  not necessarily a numerical model.
- Apply the model … within its limits.

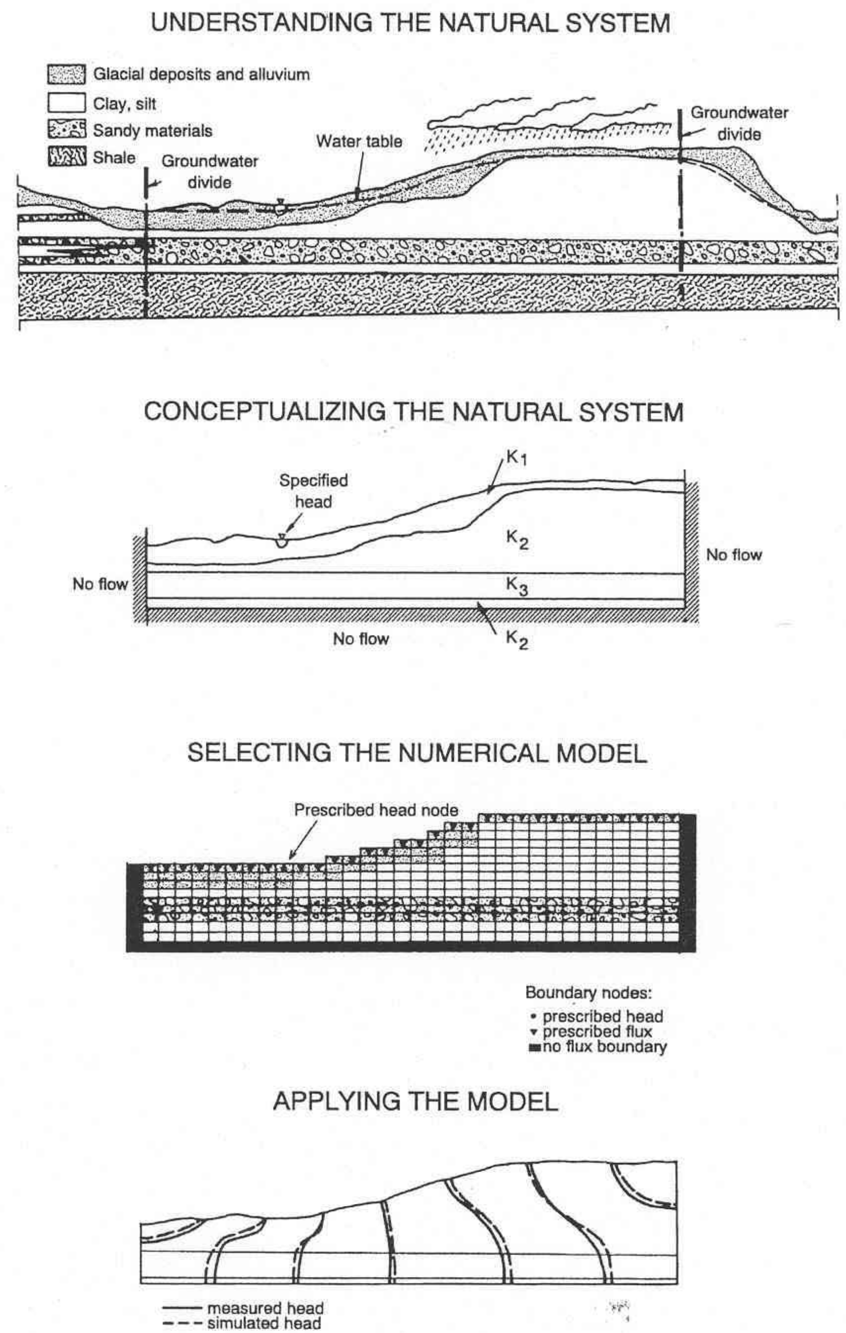
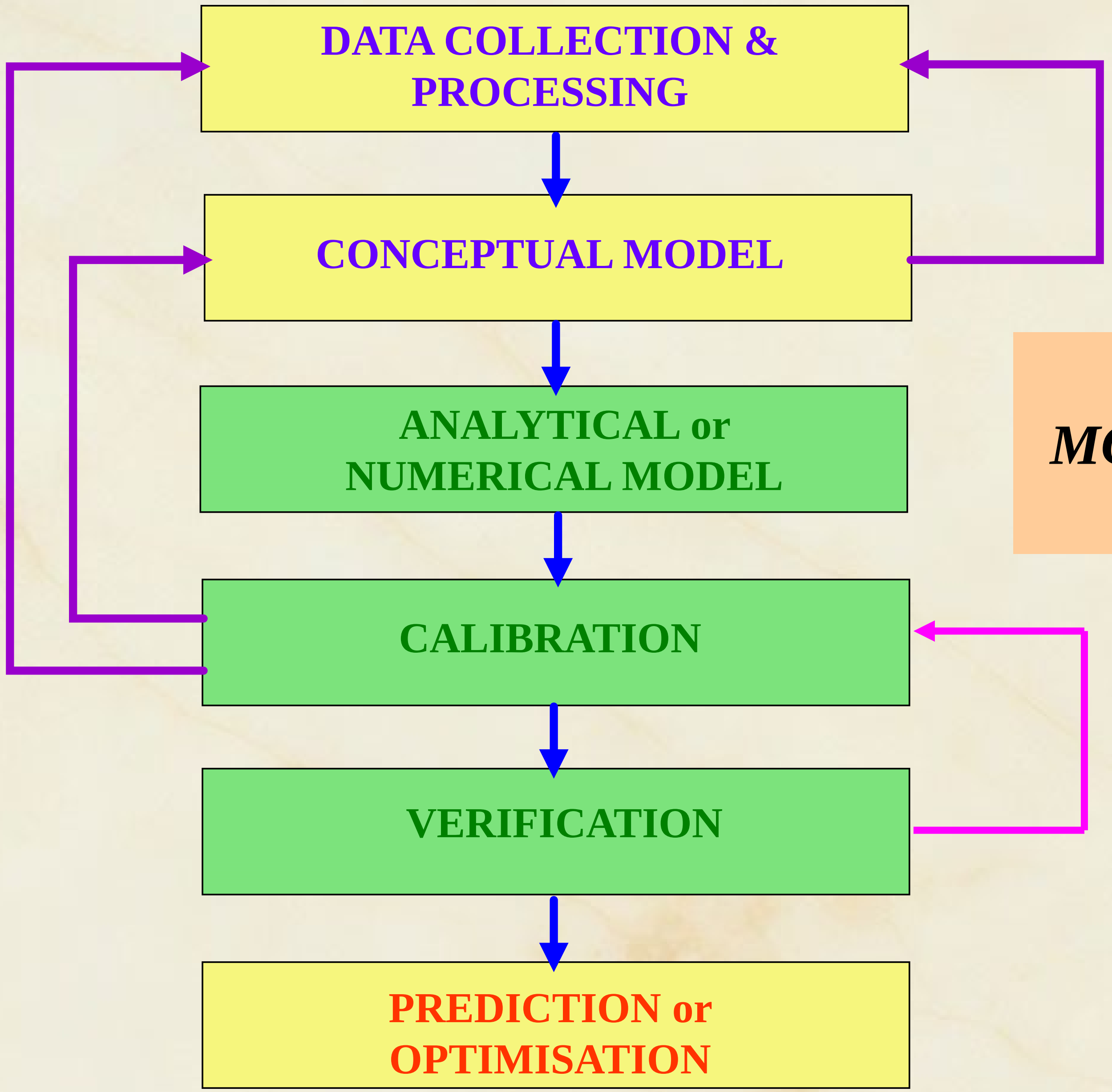


Figure 1.3 Examples of some of the main phases in model design.

Modelling Process – Flow Diagram



THE MODELLING PROCESS

Conceptual Model

Mental picture of a natural system

Idealised and simplified

Essential features to appropriate detail

Complexity depends on data, aim, budget, deadlines

can have several conceptual models for the one site

Will highlight data gaps and deficiencies in knowledge

Always ***sketch*** the picture

section or block

include major recharge and discharge processes

Conceptual Model

- A vital first step
- Up to 50% of modelling project
- Simplified representation of key features of physical system & hydrological behavior
- Design as simple as possible (*principle of parsimony*)

Types of Model

- Conceptual Model
  - *Not mathematical*
- Simple Model
  - *Low complexity*
- Impact Assessment Model
  - *Moderate complexity*
- Aquifer Simulator
  - *High complexity*

Low Complexity

- Suitable for preliminary assessment (*rough calculations*)
- Suitable for feasibility
- Minimal resources required
- Not suitable for complex conditions or detailed resource assessment
- Usually an **ANALYTICAL** model

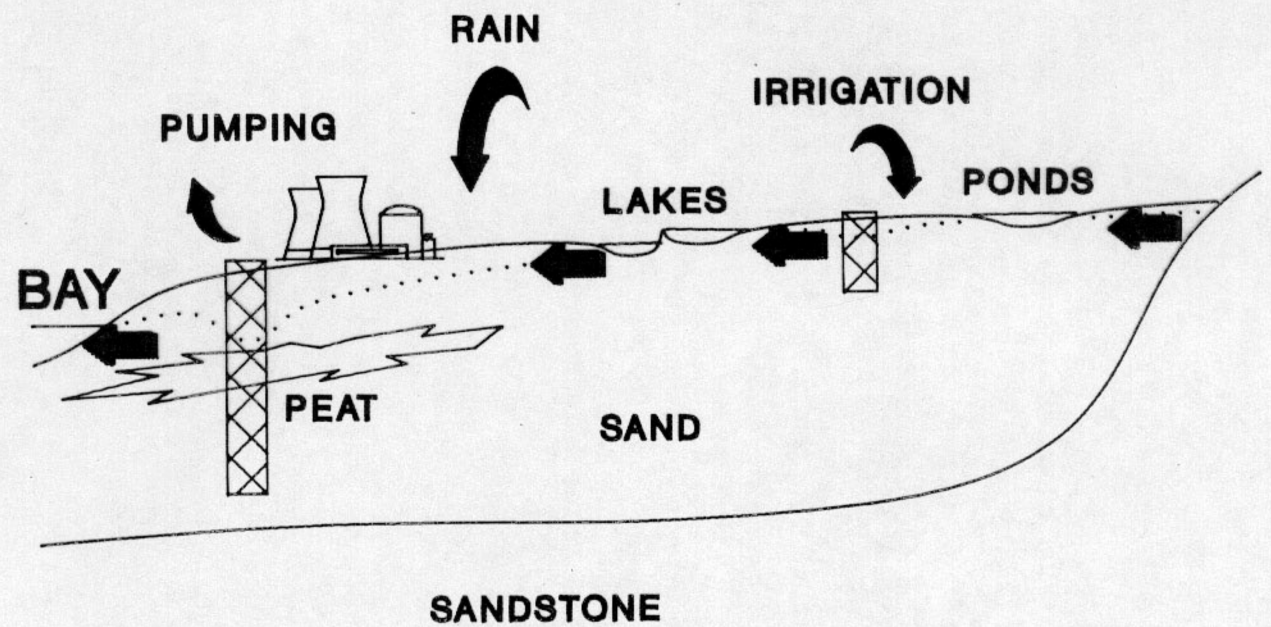
Moderate Complexity

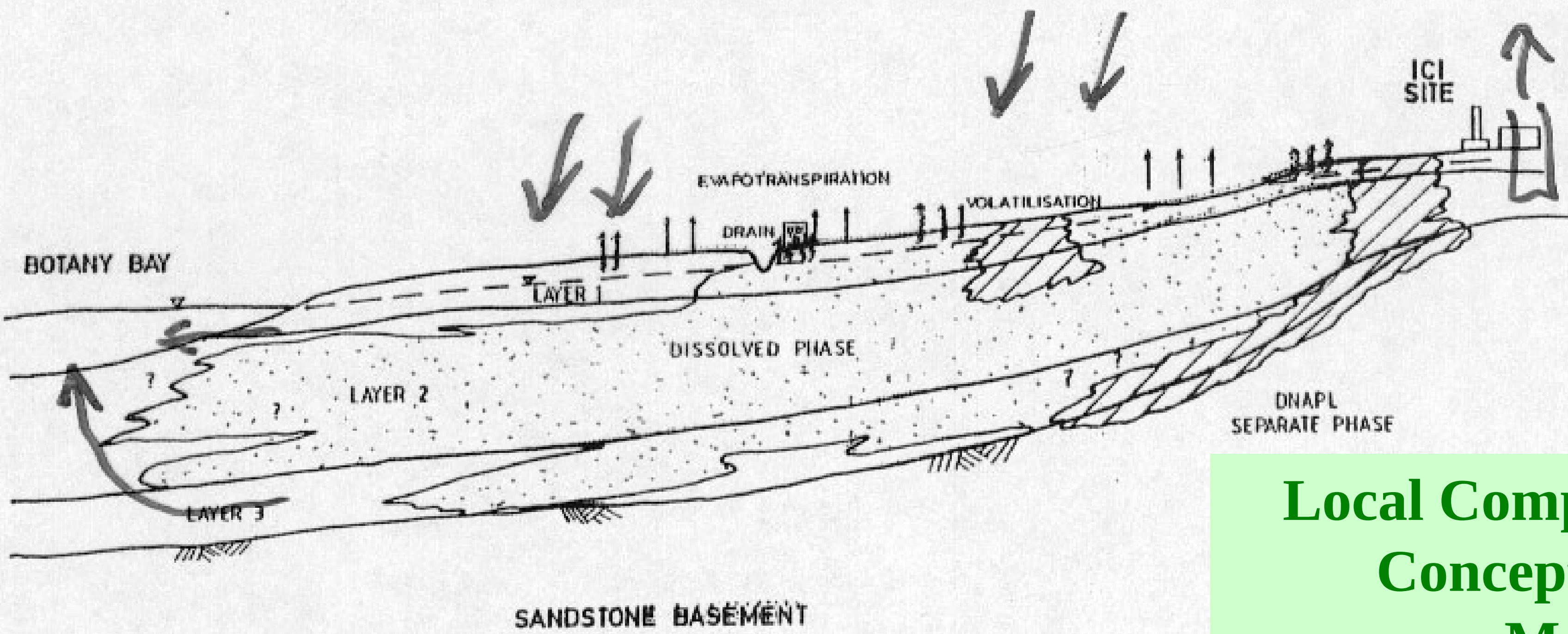
- Requires more data, sound conceptual model and sound understanding of groundwater system dynamics, recharge, discharge, etc.
- Suitable for predicting the impacts of proposed developments
- Either **ANALYTICAL** or **NUMERICAL**

High Complexity

- Suitable for predicting responses to any change in hydrological conditions
- Spatial variability, system dynamics
- Suitable for developing sustainable resource management policies
- Open-ended budgets and long time frames for ongoing development
- **NUMERICAL** model

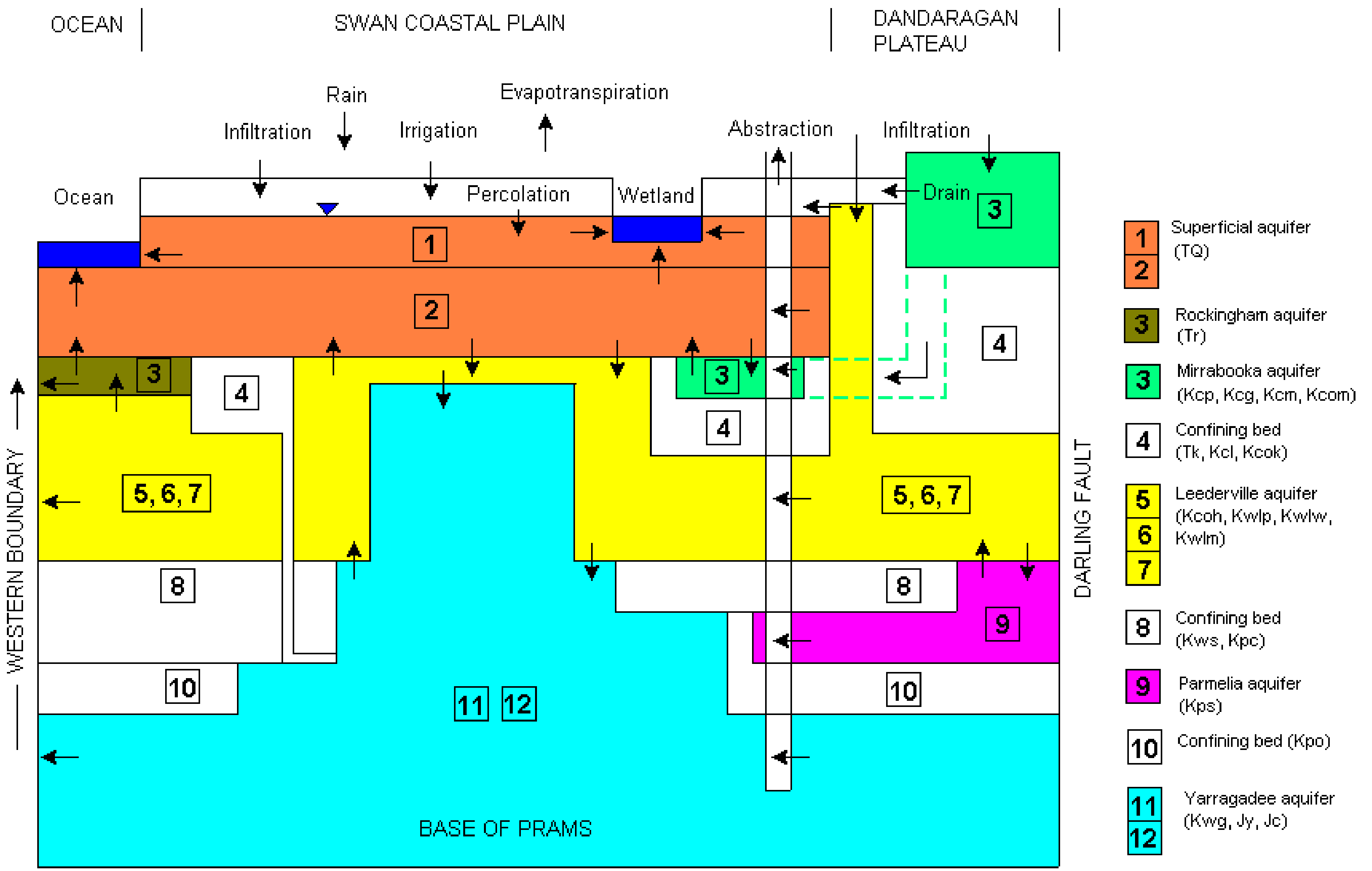
Regional Simple Conceptual Model





Local Complex Conceptual Model

Regional Aquifer System, Highly Complex Conceptual Model - PRAMS



Conceptualisation Questions [1]

- How many model layers?

  - *gross lithology*
  - *multilevel piezometers*
  - *purpose of model*
  - *section or areal model*
  - *constraints on project*
  - *analytical or numerical*
  - *aquitard handling*

- What hydrological stresses are active?

  - *rainfall*
  - *streams*
  - *floods*
  - *irrigation*
  - *abstraction*
  - *evapotranspiration*
  - *drains*

Conceptualisation Questions [2]

- is rainfall recharge important?

  - event responses in short term (groundwater rises in hydrographs)

  - residual mass correlation in long term

  - soil type

  - differential salinity

- is stream interaction important?

  - event responses in short term (compare stream and groundwater hydrographs)

  - residual mass correlation in long term

  - contour bulges/kinks

  - low groundwater salinity

Conceptualisation Questions [3]

- is flood recharge important?

  - event responses in short term (groundwater rises in hydrographs)

  - contour maps of residuals (volume estimates)

  - flood extent and duration

  - groundwater salinity map

  - soil type

- is abstraction important?

  - history of use

  - bore density

  - drawdown/recovery on groundwater hydrographs

  - which layers?

Conceptualisation Questions [4]

- is evapotranspiration important?
  - *- depth to water table*
  - *- vegetation type*
  - *- vegetation health*
  - *- evidence of salinisation*
  - *- discharge areas*

- is drainage important?
  - *- drain network*
  - *- drain elevation in relation to groundwater level*
  - *- observable seepage faces (salt line)*

ANALYTICAL MODELS

- For

  - Fast, good for feasibility studies.

  - Cheap.

  - More easily understood than numerical models.

  - Sensitivity analysis and optimisation are simple.

  - Good where data is deficient.

- Against

  - Boundaries are ignored.

  - Cannot account for spatial variability in parameters.

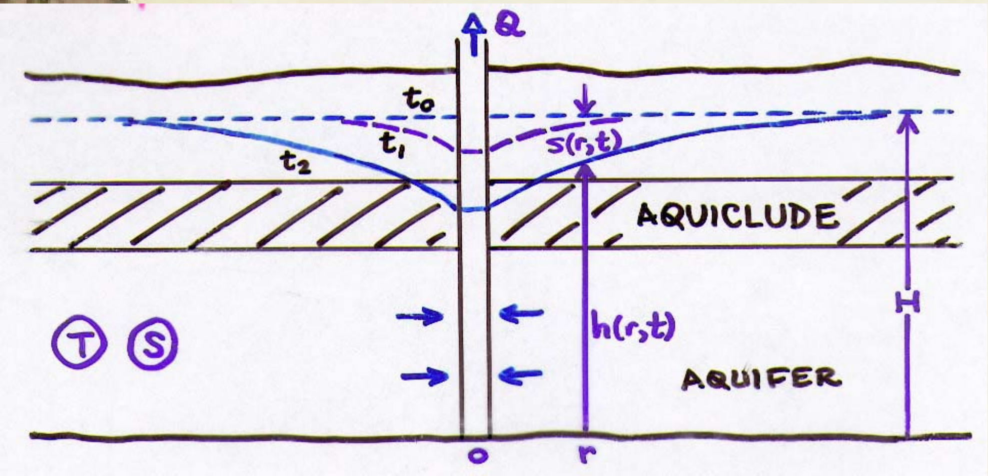
  - Very few available for solute transport.

  - Available software is quite simplistic.

  - Not defensible in court.

Theis Model

CONCEPTUAL MODEL:



ASSUMPTIONS:

- No recharge to aquifer
- Infinite horizontal aquifer (confined)
- Uniform transmissivity T
- Uniform storage coefficient S
- Constant pumping rate Q
- Fully penetrating bore

ANALYTICAL MODEL:

$$S\frac{\partial h}{\partial t} = \frac{T}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) + Q$$

subject to: $$\begin{aligned} h &= H \ (t = 0) \\ h &= H \ (r \to \infty) \end{aligned}$$

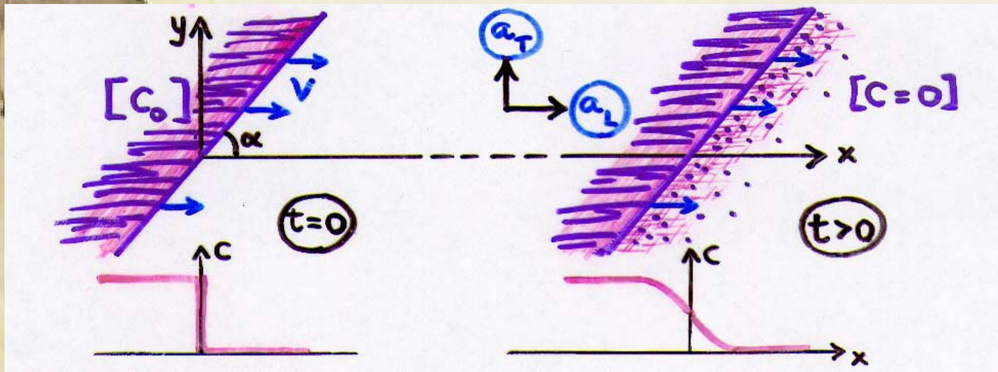
SOLUTION:

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

where: $u = \dfrac{r^2 S}{4Tt}$, $\quad W(u) = \int_u^\infty \dfrac{1}{u} e^{-u} du$

2D – Linear Solute Transport Model

CONCEPTUAL MODEL:



ASSUMPTIONS:

- No recharge
- Infinite horizontal aquifer (unconfined or confined)
- Steady 2D flow (constant velocity V)
- Uniform dispersivities α_L, α_T
- Uniform initial concentration C_0 (behind front)
- Front makes angle α with x-axis

ANALYTICAL MODEL:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x}$$

subject to:

$$\left.\begin{array}{ll} C = C_0 & (left\ \ of\ \ front) \\ C = 0 & (right\ \ of\ \ front) \end{array}\right\} t = 0$$

$$\left.\begin{array}{ll} C = C_0 & (x = -\infty,\ y = +\infty) \\ C = 0 & (x = +\infty,\ y = -\infty) \end{array}\right\} t > 0$$

SOLUTION:

$$C(x,y,t) = \frac{C_0}{2} erfc\left[\frac{(x - Vt)\sin\alpha - y\cos\alpha}{2\sqrt{(D_L \sin^2\alpha + D_T \cos^2\alpha)t}}\right]$$

where: $D_L = a_L V + D_d$, $\quad D_T = a_T V + D_d$

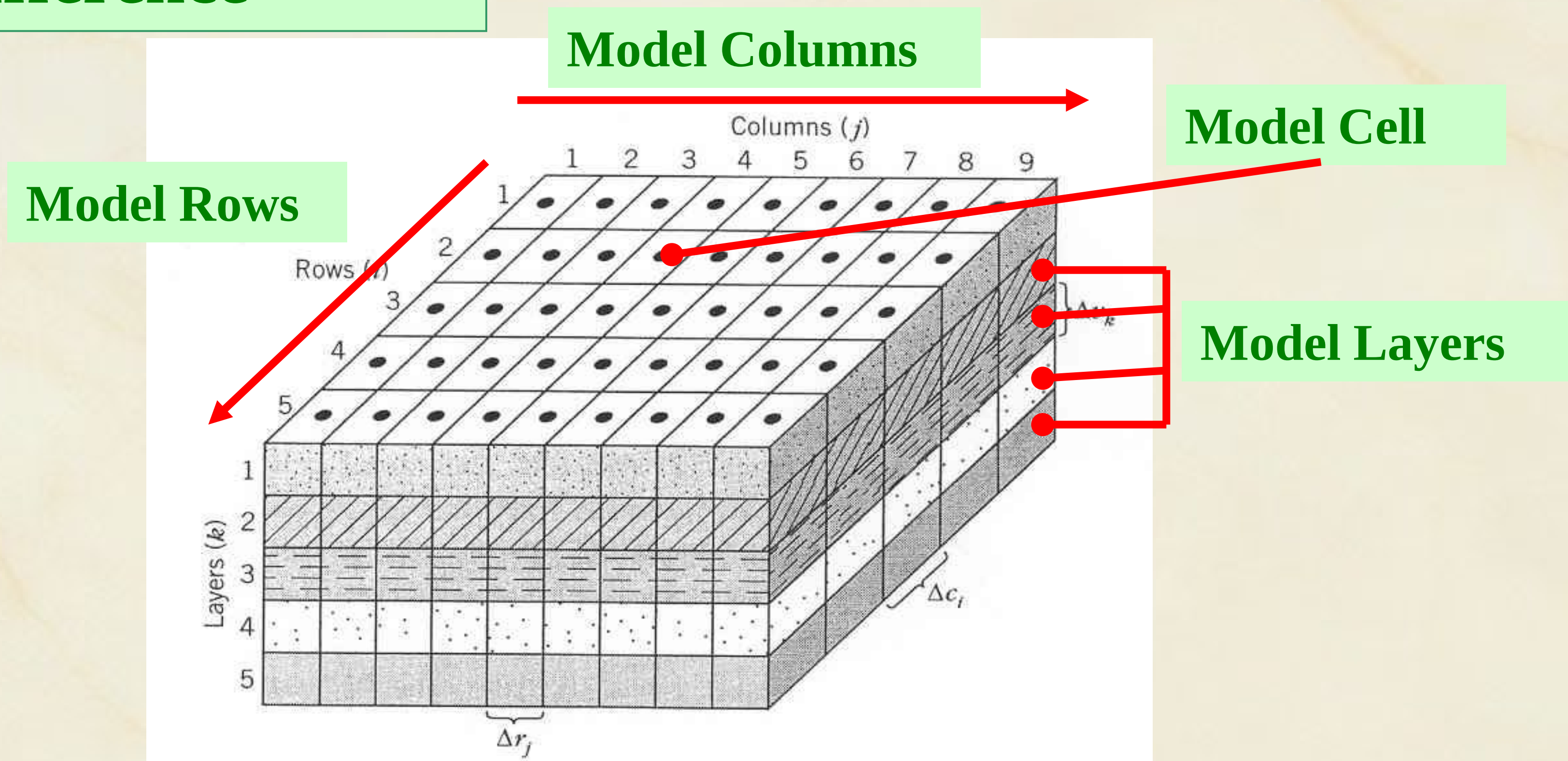
NUMERICAL MODELS

- Two Main Types

  - *Finite Difference*

  - *Finite Element*

DISCRETISATION – Finite Difference

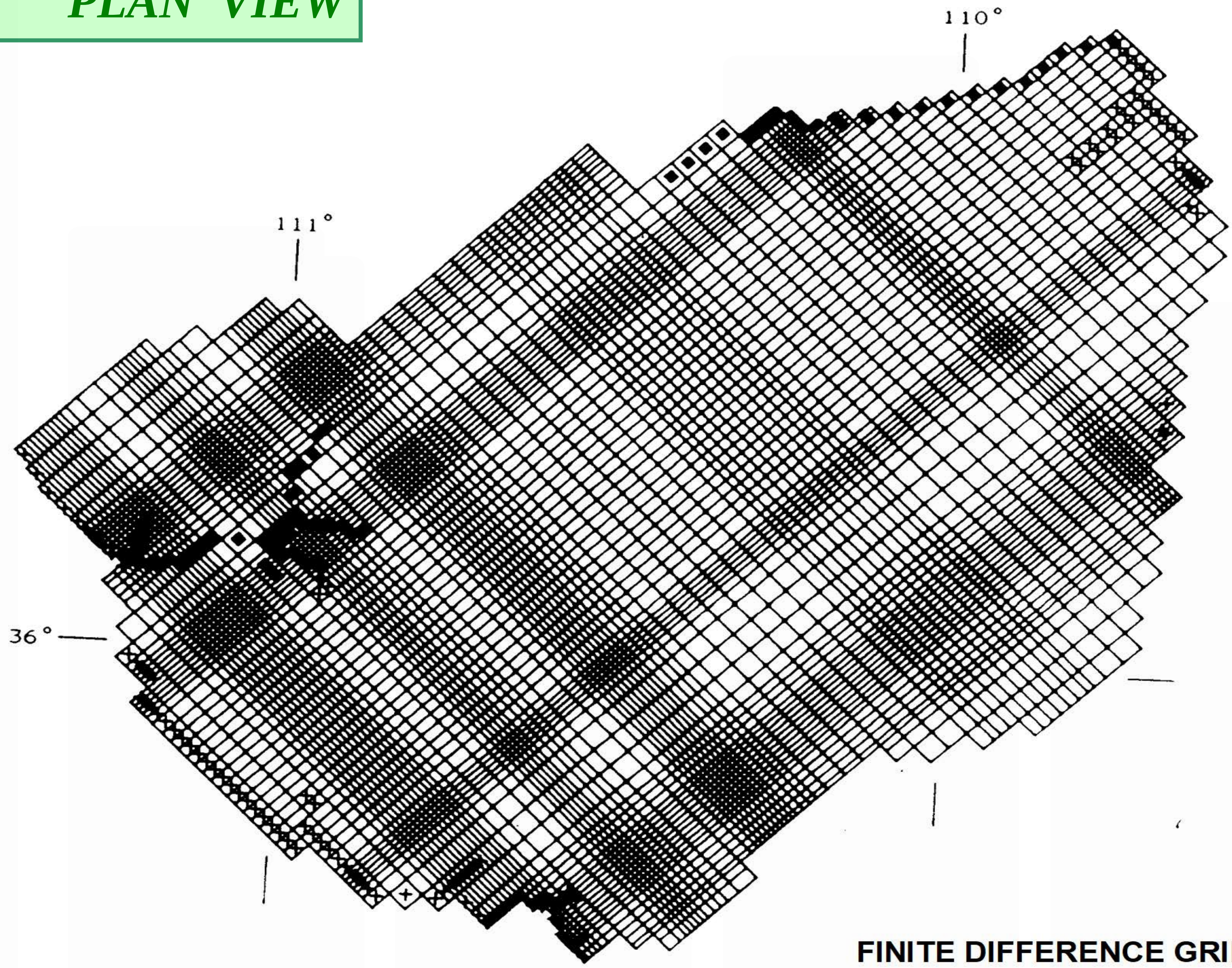


Explanation

- ● Node
- Δr_j Dimension of cell along the row direction. Subscript (j) indicates the number of the column
- Δc_i Dimension of cell along the column direction. Subscript (i) indicates the number of the row
- Δv_k Dimension of the cell along the vertical direction. Subscript (k) indicates the number of the layer

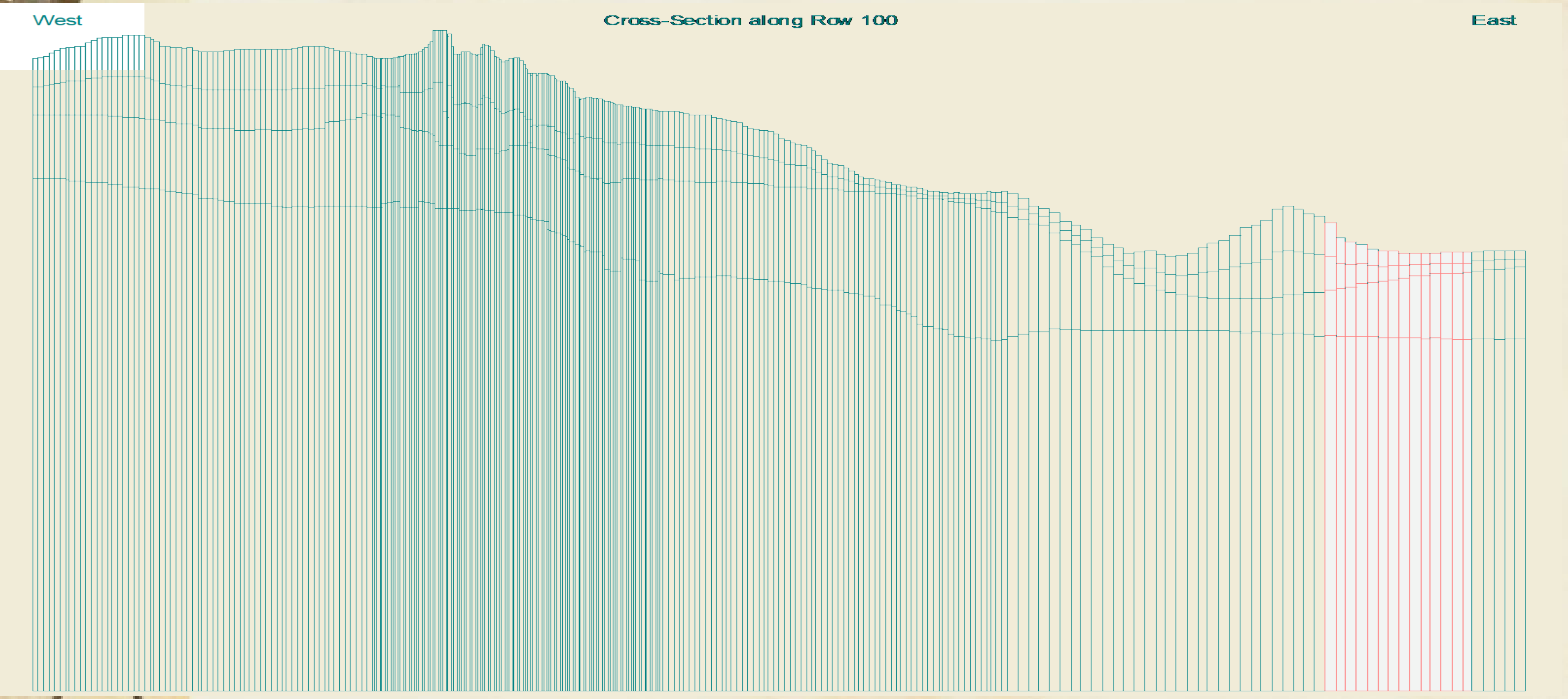
Figure 7.8 Discretization of a three-dimensional system (modified from McDonald and Harbaugh, 1988).

PLAN VIEW



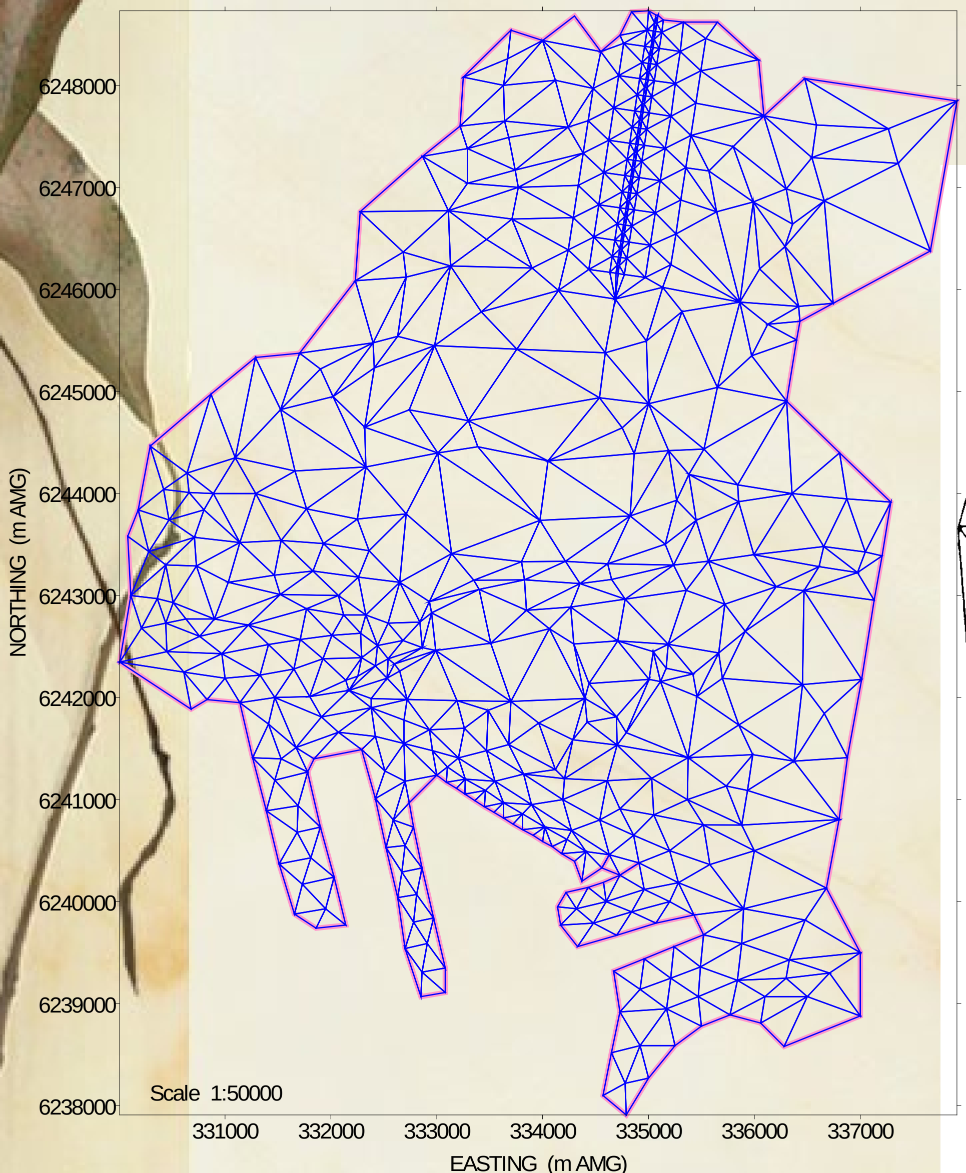
FINITE DIFFERENCE GRID

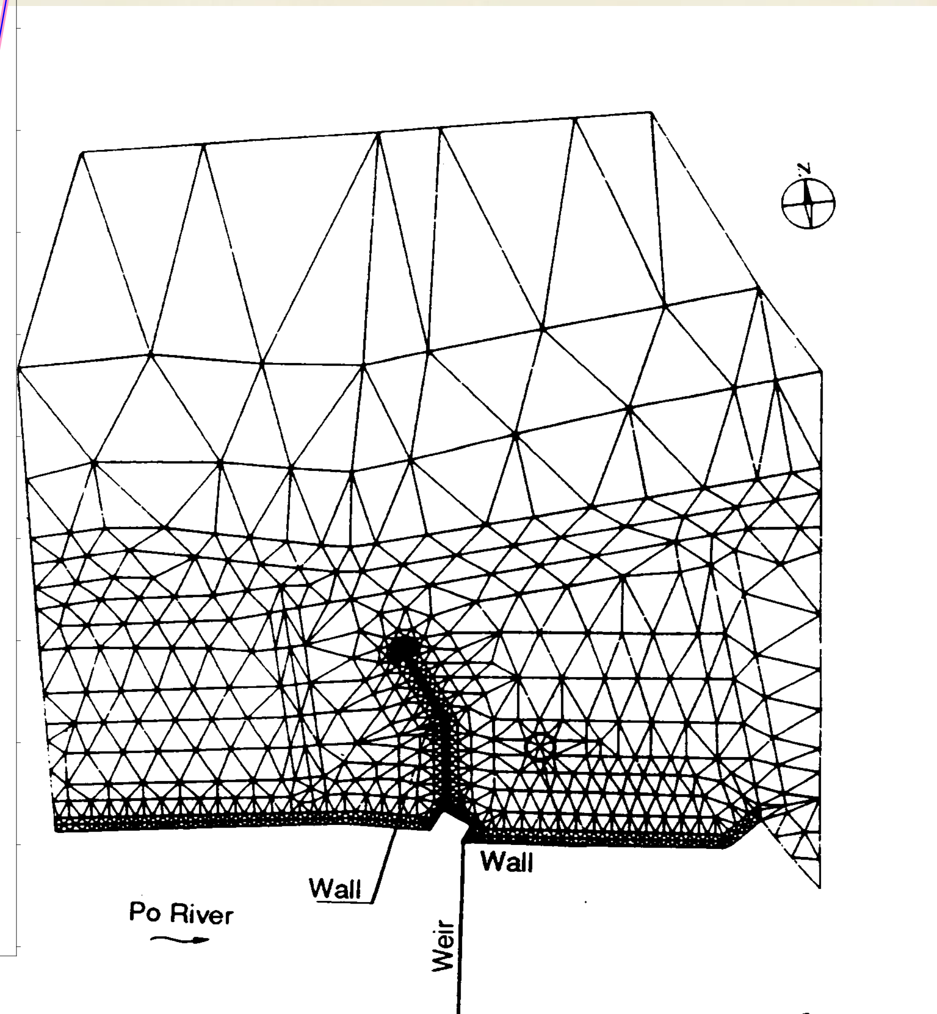
SECTION VIEW



DISCRETISATION – Finite Element

FINITE ELEMENT MESHES





Boundary Conditions

- Three Boundary Condition Types;

  - *First Type*
  - *Second Type*
  - *Third Type*

All other boundaries are variations of these three.

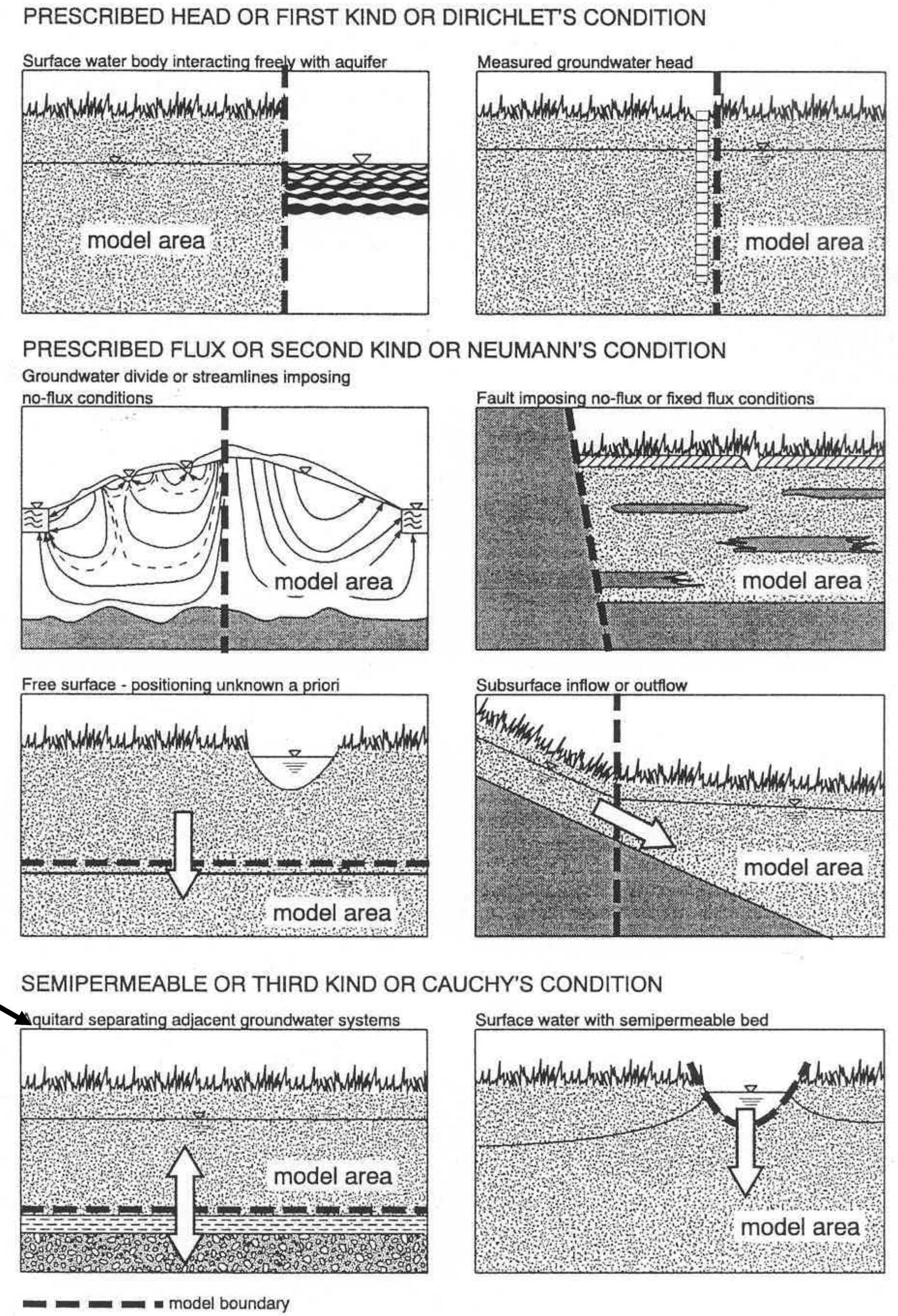


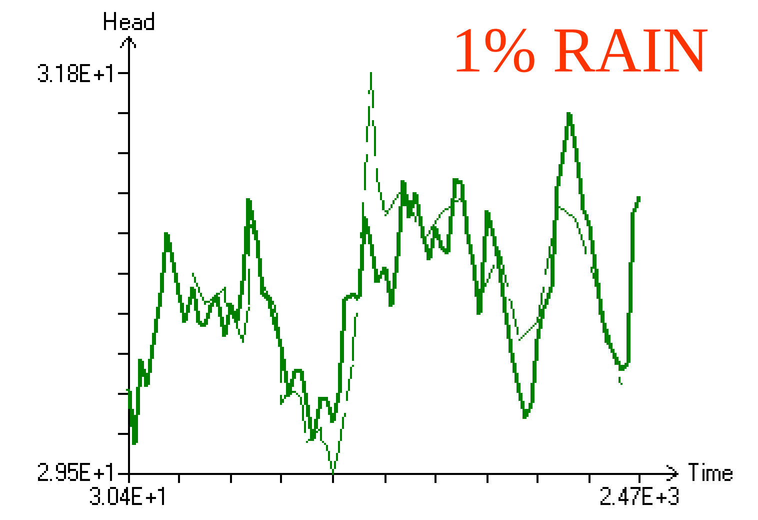
Figure 2.6 Boundary conditions in flow modeling.

CALIBRATION

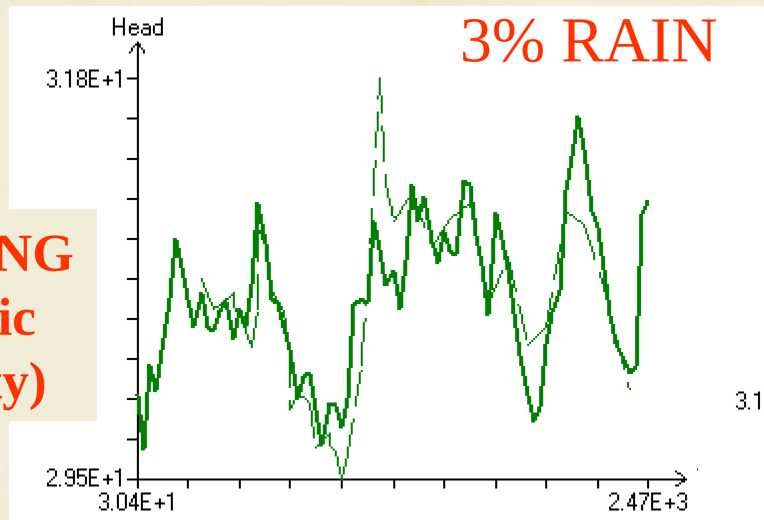
- Assume parameter distributions
- Trial and error variation
  - *later, automated inversion* (e.g. PEST)
- Match field and model data
  - *heads, drawdowns, flows, concentrations*
- Initial steady state calibration
  - *excluding storage terms*
- Transient calibration
  - *"key" hydrographs, contour maps*
- Best calibration at stressed locations
- We need to be very careful as invalid models can be successfully calibrated.

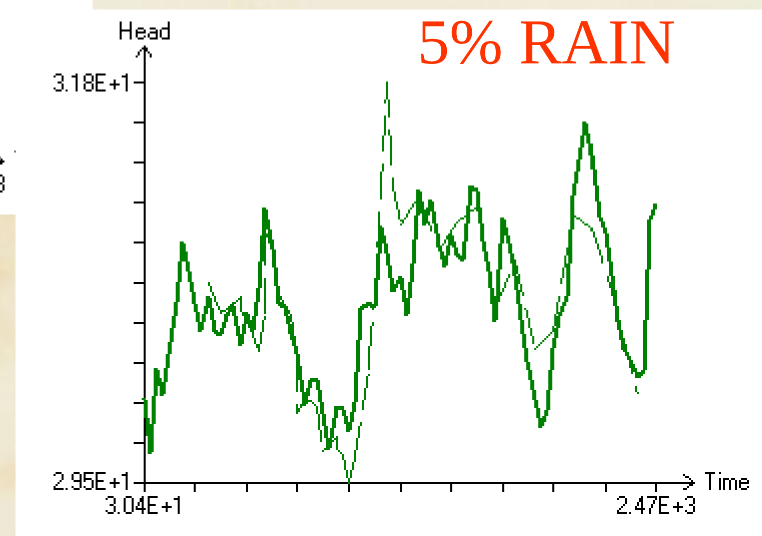
AUTOMATED CALIBRATION

Ratio Correlation:
Rain Recharge and Hydraulic Conductivity
DANGER DANGER!!
Non Unique Solution!



INCREASING K (Hydraulic Conductivity)





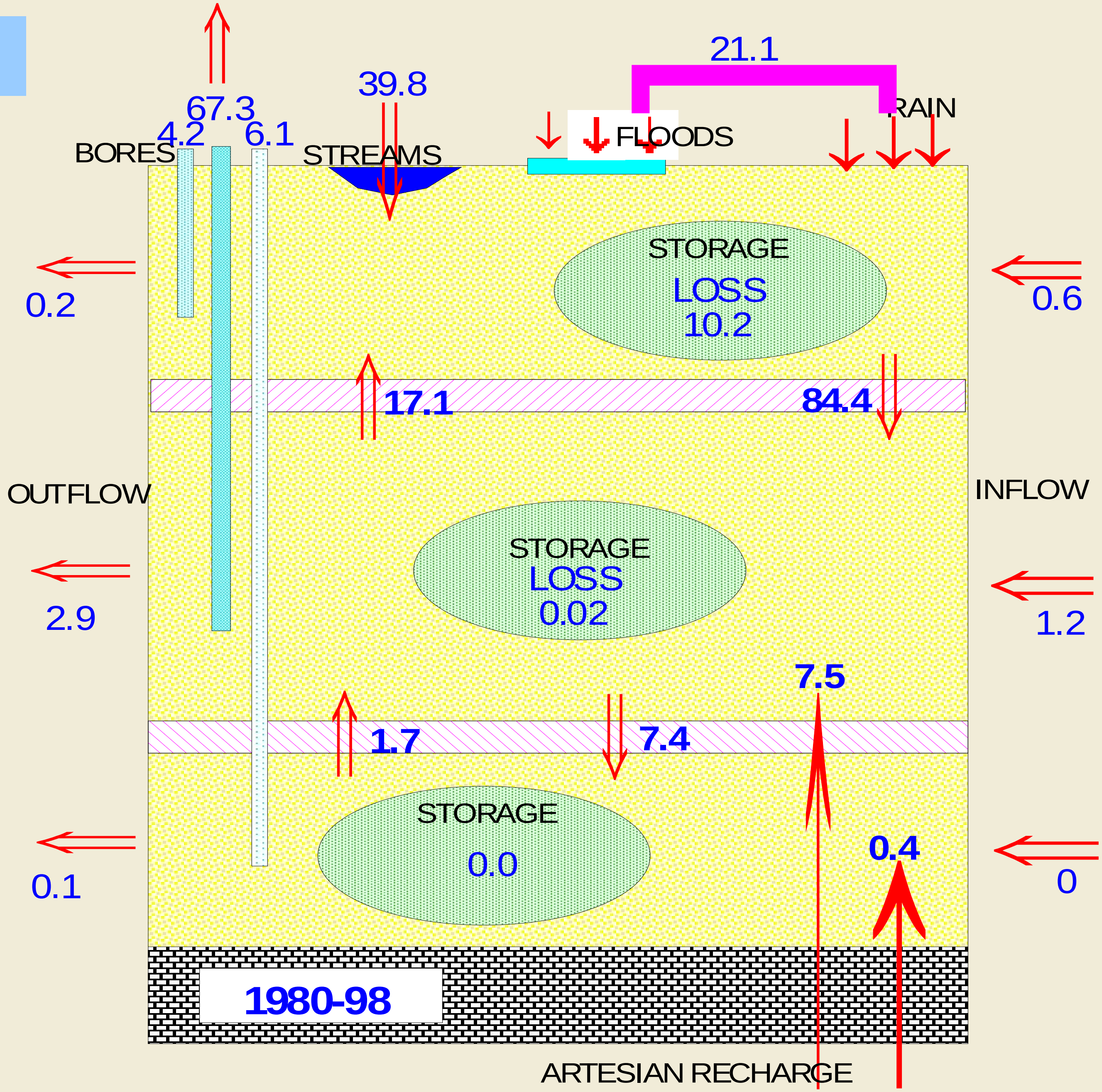
VERIFICATION

- Only for transient models
- Reserve some of the time series dataset (~ 25%)
  - *don't use it for calibration*
- Or, use full time series but reserve some spatial monitoring points
  - *"blind" hydrographs*
- After calibration, check model performance on reserved datasets

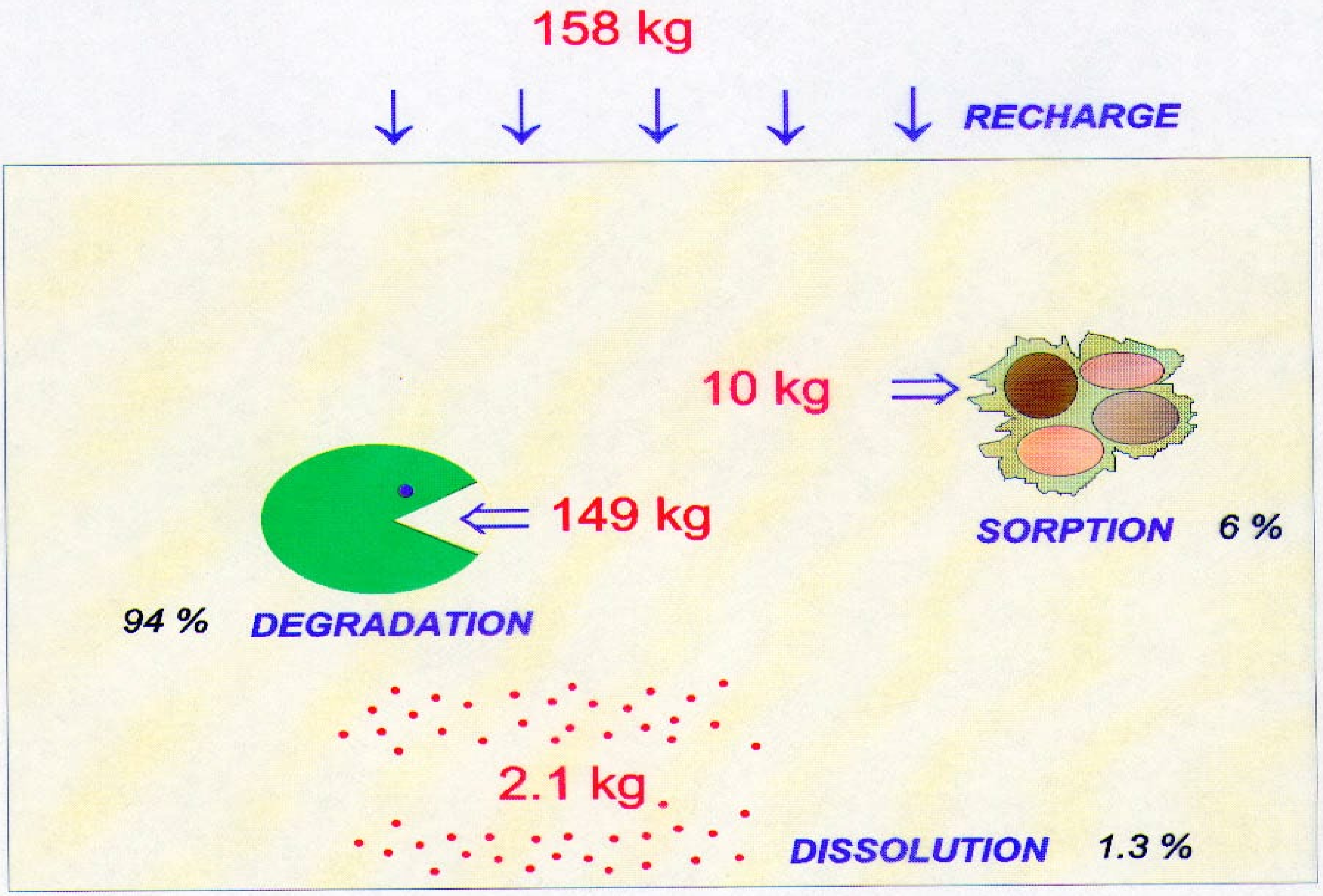
WATER or MASS BUDGET

- Very important output of a model
- Always check for physical reasonableness
- Present as a table, or as a schematic
  - *re-use the conceptual model sketch*
- Software allows sub-regional budgets

Water Budget



Mass Budget for Benzene (40 years)



-1.3 % MASS BALANCE ERROR: -3.1 kg

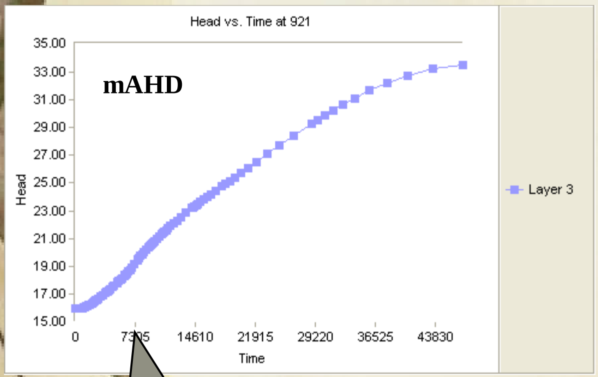
PREDICTION

- Steady state or transient
- Steady state
  - *use range of hydraulic stresses to indicate uncertainty*
- Transient
  - *use historical record; cycled historical record; synthetic hydraulic stresses*
- Scenario analysis or stochastic simulation

Water, Levels & Flow, Water Quality – Solute Transport Predictions, concentrations and travel times.

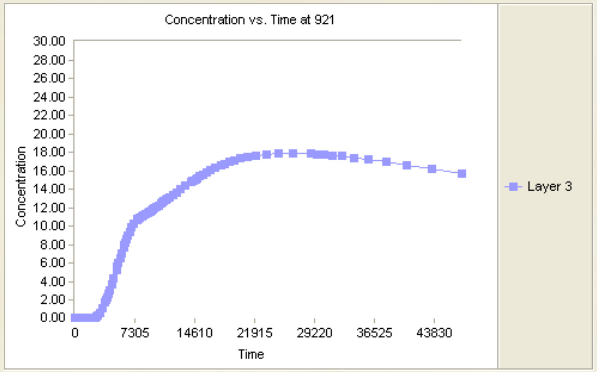
SIMULATED CHEMICALS

130 years

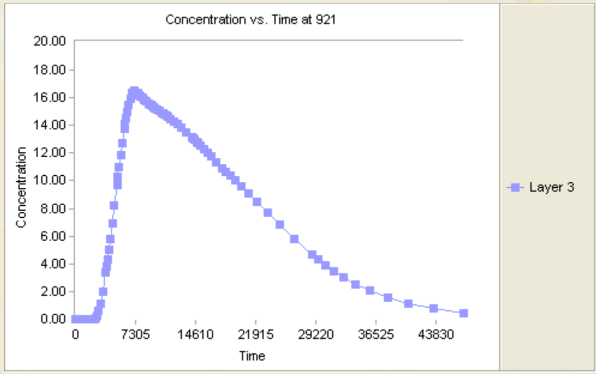


20 yr tics

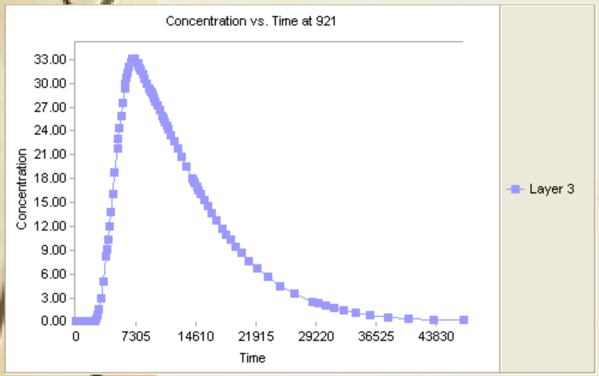
Water Level



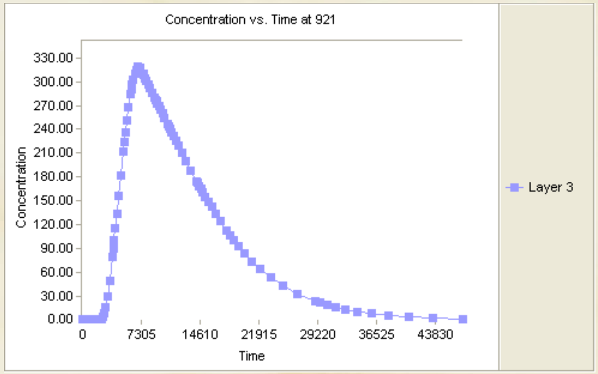
TDS (%)



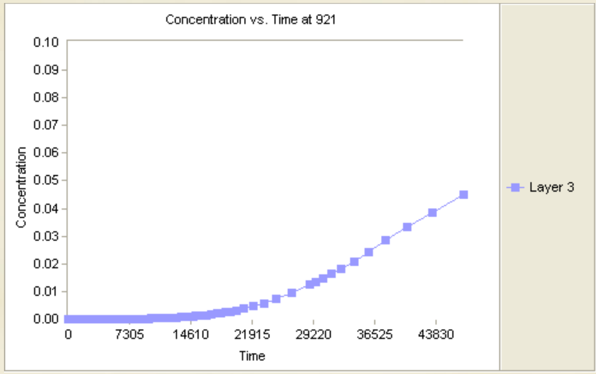
Toluene (ug/L)



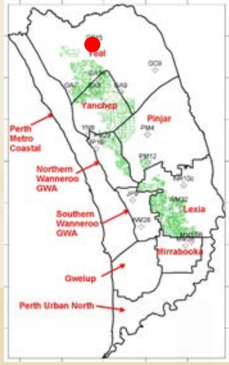
As (ug/L)



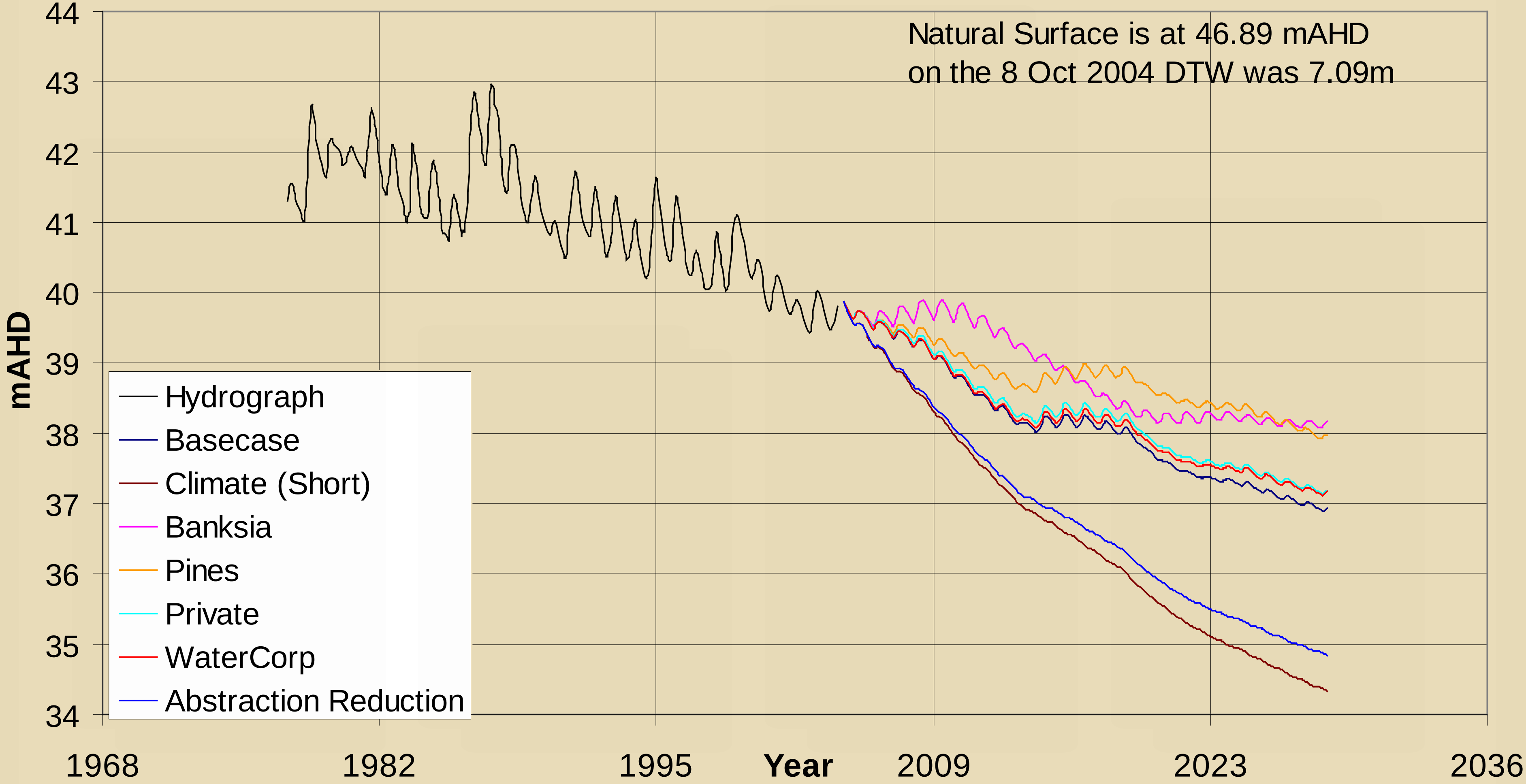
Fe (mg/L)



Zinc (mg/L)



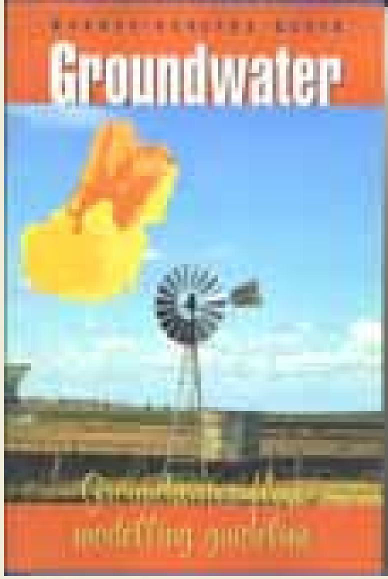
Observed and Predictive Hydrograph GB15



OPTIMISATION

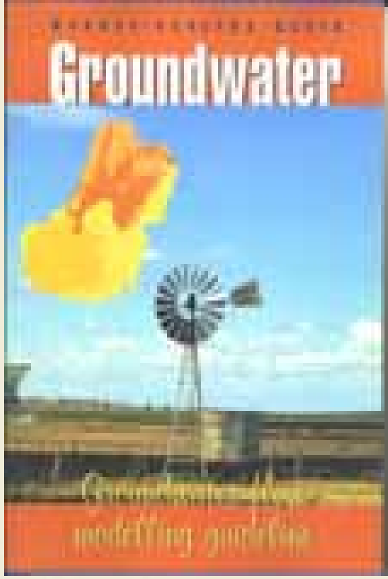
- Powerful way to add value to an existing model
- Impose water level limits & constraints on pumping (e.g. entitlements)
- Determine optimal pumping distribution and schedules that satisfy all constraints
- OPTIMAQ proprietary software
- MODOFC
  - *included in some Modflow interfaces*

Groundwater Flow Modelling Guidelines



- Sponsored by MDBC (2000)
- Prepared by *Hugh Middlemis*, *Noel Merrick* & *John Ross*
- To promote transparency, consistency, best practice
- To raise the minimum standard of practice
- To de-mystify modelling for clients, community, and non-specialists
- Not intended to limit creativity required, or deny experienced professional judgment

Groundwater Flow Modelling Guidelines



67 guidelines in 27 categories

- Modelling best practice
- Objectives, complexity, data collation, units
- Conceptual model, code selection
- Model study plan
- Model construction, initial conditions, calibration, non-uniqueness, performance measures, verification
- Prediction, sensitivity analysis, uncertainty analysis
- Reporting, archiving, Review, audit

If your considering getting into groundwater modelling my advice is …

DON'T!!

you will be in the firing line every day!! It is however a very powerful tool, the most powerful that we have for managing groundwater.

But models need to but constructed and used correctly or their results can be very misleading, forewarned is forearmed!! You can't trust a model only a modeller.