Stem Taper and Volume in *Pinus pinaster*

T.B. Butcher 1979

Introduction

Pinus pinaster tree and stem volumes are determined locally using the following volume formula provided by Campbell and Byrnes (pers com) -

$$Vol(h)=[(22(D^2)/280000][(H_t-H_{st})^Q-(H_t-h)^Q]/[Q(H_t-H_b)^{Q-1}]$$

where

$Q = 2\alpha + 1$
H_t = total height (m)
h = height up stem (m)
H_{st} = stump height (m)
H_b = breast height (m)
D = diameter under bark at breast height (cm)
$Vol(h)$ = merchantable volume under bark to height h

The formula, based on a model used by Ormerod (1973), is derived from the flexible geometric model of

$$d = D((H_t-h)/(H_t-H_b))^\alpha \quad , \alpha > 0.$$

with

d = upper stem (merchantable) diameter (cm)

Ormerod recommended this general stem profile as a suitable basis for developing volume equations for inventory and for research. The equation is conditioned that when $h = H_b$, $d = D$, and when $h = H_t$, $d = 0$. If the fitted exponent α is less than one the shape of the whole will be parabolic and if greater than one, then neiloid. An alpha value of unity is for a conic tree profile.

Ormerod derived the following model to estimate stem height

$$h = H_t-((d/D)^{1/\alpha} \, (H_t-H_b)$$

To counter for the changes in form along the bole length a modified form of the stem profile model as a step function was suggested. This was not adopted in initial testing locally where butt swell was assumed to be insignificant. Sufficiently accurate estimates were available without using the step model and the additional requirement for measurement of the diameters and heights of the sectional intercepts.

The stem profile equation has also been used by Bruce et al (1968) and by Reed and Byrne (1985) who developed equations for total volume and volume ratio through integration of the model. An advantage of the model is that it has only one parameter to estimate and is simple in use in practice. It has been pointed out however, that the taper and volume models are not compatible. Martin (1981) included the model in tests for Applalachian Hardwoods and found it to perform poorest, of the several models tested, for bias in diameter, height and volume prediction. Contrary to this Martin (1984) ranked the model highly in predicting log volumes accurately but found that the Ormerod equation performed badly in merchantable volume estimation when heights had to be estimated from this form of the model. Cao et al (1980) considered the Ormerod model to be a good non-linear taper equation. Integration of the diameter equation, however, did not provide good estimates of merchantable volume.

The alpha values provided locally for the volume equation varied from 0.62 to 0.71 for *Pinus radiata*, depending on the plantation source. A single value of 0.49 was provided for *P. pinaster*: presumably derived from sample trees in plantations north of Perth.

Full volume for local *P. pinaster* trees has been calculated from DBHub and Tree Height using either the Ormerod Equation or a Constant Form Factor (Clutter et al, 1982) model of

$$\text{Vol (ub)} = cD^2H_t(22/280000)$$

where c = 0.5659

It was believed that both the alpha value and form factor was calculated for *P. pinaster* using data from a stem analysis in South Lane Poole Block, Gnangara plantation.

For comparing treatment performances in thinning and fertiliser trials for *P. pinaster* north of Perth the alpha value of 0.49, or the form factor value of 0.566 was used in all cases. This is satisfactory for experimental purposes. The accuracy of these values require definition if volumes are to be related to commercial use rather than for comparing the performance of different experimental treatments.

Sample Data.

Six sets of sample tree data were obtained for comparison.

1. South Lane Poole Block, Gnangara Plantation, planted in 1943, at 1.8 x 1.8 metre square spacing. Superphosphate was applied at 60 g per tree at time of planting and in 1945. At age 20, 500 kg ha^{-1} of superphosphate was broadcast in the stand. Thinning was at age 18 years to 750 s ha^{-1} and at age 27 to 500 s ha^{-1}. Pruning to 2.1 m was at age 7 years. Site index ranged from 23 - 26 m at age 30 years, placing the stands in the top site quality for the plantation.

A series of 10 plots was established to determine the later development of stands for a second rotation trial (WP 29/71). Six of these plots were measured sectionally by climbing over the period 1974-1979. Diameters over bark and four times bark thickness were measured at stem heights 1.3 1.5, 3.0, 4.5, 6.0, 7.5 9.0 and 10.5 m. Total Height and height to green crown were also measured. Forty trees from the 1983 measurement of plots 5 and 9 and 39 trees from the 1974 measurement of plots 3 and 4 were analysed for total volume and volume to each stem height. Sectional measurements did not progress sufficiently high on the stems to obtain a realistic sectional measurement. To overcome this deficiency the taper model of Kozac et al (1969)

$$d^2/D^2 = -1.04(h/H_t - 1) - 0.0025(h^2/H_t^2 - 1)$$

was fitted to the data and diameters under bark were estimated for stem heights 13.5, 16.5 and 19.5 m to determine total volume under bark for each tree. The data sets for the 39 and 31 year old ages were separated as Sample Groups 1 and 2. Following initial examination these were combined into a SLP Data Set (Table 1).

Table 1. Summary of sample tree data sets availability for testing stem profile and volume attributes.

1. South Lane Poole

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DBHub	69	31.5	32.6	6.507	0.783	20.1	44.6
Height	69	23.3	23.8	1.929	0.232	19.5	27.9
Volume	69	0.98	0.99	0.4655	0.056	0.31	2.12

2. Early

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DBHub	68	17.1	16.7	5.933	0.710	6.4	30.5
Height	68	17.3	16.3	3.164	0.384	11.4	24.1
Volume	68	0.24	0.19	0.193	0.023	0.02	0.82

3. Ritson

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DUB	119	23.2	21.1	7.636	0.644	11.3	46.9
Height	119	12.7	12.3	3.573	0.313	6.7	23.2
Volume	119	0.32	0.21	0.287	0.024	0.06	1.34

4. Stem analysis

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DBHub	31	23.8	23.3	2.074	0.373	20.9	28.7
Height	31	21.1	21.1	0.611	0.110	19.8	22.5
Volume	31	0.52	0.49	0.094	0.017	0.38	0.77
Age	31						

5. Ritson Stem Analysis

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DBHub	113	18.9	18.6	5.984	0.564	4.65	34.6
Height	113	12.5	12.2	3.621	0.341	3.70	21.0
Volume	113	0.19	0.15	0.151	0.014	0.00	0.83

2. Sample Group 2 was a series of 127 sample trees collected in 1959 from a range of local plantations and stand age classes and included trees in the 1942 planting in South Lane Poole Block at age 17 years. Sixty eight of these had sectional measurements to at least 80 percent of total height and were

used for total volume estimation. These were referred to as the Early Sample Data (Table 1).

3. A third data set was available from measurements being undertaken by P. Ritson from *P. pinaster* growing on farms in the south west region. Initially 211 trees were available but only the 119 (Table 1) with sectional measurements up to at least 80 percent of tree height were examined in detail.

4. A fourth data set arose from stem analysis of trees felled in the second rotation study at Gnangara. In March 1972, 40, thirty year old *Pinus pinaster* trees in South Lane Poole, Block A76, at Gnangara were sectioned for stem analysis. The trees were in Blocks 2 and 3 of the Working Plan 29/71 studying the performance of a second rotation of pines on one of the oldest stands of Portuguese pinaster available. Stem analysis was to document past growth on the site by comparing predominant height (height of the tallest 254 trees ha^{-1}) with that of the future rotation. It is believed that these trees were used by Neil Sumner (pers com) to obtain an alpha value for the species. He also used the data set to develop a segmental polynomial volume formula (Max and Burkhart, 1976) for the species. Thirty one of these trees were used, the other nine had suspect measurements which could not be verified.

5. The fifth data set consisted of 113 farm trees measured separate to those in data set 3 but over the same plot range. These trees were subject to stem analysis.

Table 2. Validation of 8 full volume equations fitted to three data sets. All derivations are from weighted regression. The number (N) refers to the trees separated from the data set for validation purposes. Deviations are expressed as a percentage of the difference between predicted from observed values.

Variable	Combined Data			SLP Stem			Ritson Stem		
	N	Mean	SD	N	Mean	SD	N	Mean	SD
CFF	103	3.1	9.6	265	0.5	4.4	112	5.8	21.1
CV	103	0.3	8.4	265	1.7	4.9	112	2.7	25.4
GCV	103	0.6	8.4	265	1.7	5.0	112	-3.6	26.7
Log	103	-0.9	8.1	265	1.6	5.0	112	-3.3	19.3
GL	103	0.9	8.0	265	1.0	5.1	112	-0.1	21.8
Honer	103	0.6	9.7	265	1.9	5.0	112	-1.2	23.0
Orm	103	0.8	10.1	265	-0.1	4.6	112	-4.3	19.4
Stone	103	14.5	22.6	265	16.1	4.6	112	12.2	26.7

Procedure

Full volume. - The Constant form factor (CFF), Combined variable (CV), Generalised combined variable (GCV), Logarithmic (Log), Generalised Logarithmic (GL), Honer transformed variable (Clutter et al, 1982) and Stone (Martin, 1984) full volume models were compared with the Ormerod (Orm) model. The 8 models were calculated for the Combined Data Sets 1-3, the SLP Stem Analysis Data set 4 and the Ritson Stem Analysis Data set 5. In each case half the data were used to construct the model and the other half was used for validation. Both weighted (Clutter et al, 1982) and non weighted regressions were calculated. As the weighted regression usually performed with less bias these values only are presented in Table 2. Results for Mean deviation and Standard Deviation refer to the deviation of the predicted from the actual volume, expressed as a percentage of the actual volume.

It was assumed that the Constant Form Factor, and Logarithmic were satisfactory models over the range of data sets (Table 2). These are compared with the Ormerod full volume model in each of the Early, SLP and Ritson Data sets in Table 3.

Table 3. Fit of volume equations derived from the Combined, SLP Stem Analysis and Ritson Stem Analysis data sets against the original SLP, Early and Combined data sets. The three equations derived for each data set were the Constant Form Factor, Logarithmic and Ormerod full volume. Mean and standard deviation are for differences between actual and predicted volumes expressed as a percentage of the actual volume.

Variable	Early			SLP			Ritson		
	N	Mean	StDev	N	Mean	StDev	N	Mean	StDev
CombCFF	123	6.4	7.4	34	3.8	2.7	167	-2.0	12.8
CombLog	123	1.1	7.0	34	4.9	2.9	167	-1.4	11.1
CombOrm	123	4.8	7.2	34	4.6	2.8	167	-9.2	12.8
SlpSCFF	123	-0.1	8.0	34	-2.9	2.9	167	-9.2	13.7
SlpSLog	123	-1.4	8.5	34	-6.3	2.7	167	-4.3	14.5
SlpSOrm	123	-1.0	7.6	34	-1.5	2.9	167	-15.1	13.4
RitsCFF	123	13.3	6.9	34	10.9	2.5	167	5.4	11.9
RitsLog	123	15.7	6.1	34	19.0	2.6	167	5.8	10.5
RitsOrm	123	16.2	6.3	34	16.8	2.5	167	2.1	11.8

For plantation grown trees, represented by the Early and SLP data sets, both the Combined and SLP Stem Analysis equations were acceptable in both bias and accuracy. No clear preference for one of the three models was shown. The bias, as indicated by the mean percentage deviation, was excessive for the farm derived models in validation with the plantation data sets. It was satisfactory in validation with the Ritson farm trees. The Ritson single tree data were excessively variable compared to the plantation data sets. Again no single model displayed a particular advantage over the others.

Stem Profile and Volume Models

The Ritson Stem Analysis data set was excluded from further examination. All data sets were applied to a range of stem profile and volume models (Table 4).

The volume models 4, 5, 8 and 9 fitted each data set very well. The models for merchantable 'd' and 'h' (1, 2 , 3, 6, 7) varied from excellent to good.

The alpha value of 0.49 adopted for local use of the volume formula was derived from the SLP stem analysis when the value for all trees from age 5 to 30 years was 0.49 (Q = 1.98). The value of 0.48 (Q = 1.95) for stem analysis in Table 4 relates to trees aged 12 to 30 years, selected to provide more realistic values for the various models fitted. The previously accepted alpha value of 0.49 for the Volume Formula for local pinaster is low for either plantation or farm grown trees.

Models to estimate merchantable diameter (d) and height to merchantable diameter (h) were constructed along with the volume formula. Volume ratio models were included in the study for their utility in volume table construction. The American approach appears to be to calculate the total volume of the tree from a volume formula and then determine merchantable volumes from the volume ratio formulae. Most models fitted very well with R^2 values in the high ninety percent. The best fits were obtained with the South Lane Poole data which was of uniform site and silviculture. The Ritson and the Early data sets were similar in the amount of variation explained in each case. The Ormerod Volume Formula fitted all data sets very well.

Effect of Measurement System on Total Volume

Early in the study two main sources of variation in sample data became apparent. This variation was associated with either the source of the sample data and hence relate to silviculture and stand history or to the adequacy of measurement.

Silviculture - The alpha values for the Ormerod volume equation for the plantation data sets were 0.48, 0.56 and 0.55 for the stem analysis, SLP and Early data sets, respectively. The Ritson values were quite separate and within the range 0.69-0.71. The two groups are quite distinct data sets, the former relating to mature, closely planted, plantation grown trees with seed imported from Portugal and the later from wider spaced, younger trees on farms, often from row planting and mostly from seed obtained from the local seed orchards. The different silviculture provided distinct stem profile parameters.

Larson (1963) considered that decreasing taper is associated with decreasing crown ratio, increasing age, increasing stand density, increasing site quality and decreasing DBH-total

Table 4. Stem volume and profile models tested by five sets of sample data for *Pinus pinaster*. The Combined data set is the Early plus the Ritson set. Nomenclature is as previously defined with a, b and e constants.

	S.L.Poole	Early	Ritson	Stem Analysis	Combined
Taper Models					
1.	$d^2/D^2 = a(h/H_t - 1)+b(h^2/H_t^2-1)$ (Kozac et al 1969)				
a	-0.0925	-0.9861	-2.1743	-0.9994	-1.6470
b	-0.0920	-0.0888	0.8754	-0.0682	0.4495
N	759	897	1460	5979	1582
R^2	0.9871	0.9718	0.9268	0.9691	0.9225
S	0.0338	0.0609	0.1230	0.0552	0.1108
2.	$d = D((H_t-h)/(H_t-H_b))^{\alpha}$, $\alpha > 0$. (Ormerod 1973).				
α	0.5070	0.5066	0.7313	0.5371	0.6009
N	759	897	1460	5979	1582
R^2	0.9955	0.9870	0.9653	0.9610	0.9730
S	0.6816	0.7334	1.8455	1.1562	1.6488
3.	$h = H_t-((d/D)^{1/\alpha}\ (H_t-H_b)$ (Ormerod 1973).				
α	0.5167	0.5018	0.7457	0.4814	0.5786
N	759	897	1460	5979	1582
R^2	0.9864	0.9360	0.9427	0.9681	0.9569
S	0.7847	1.1691	1.1356	0.9304	1.2397
Merchantable Volume based on Heights and Diameter (Ormerod)					
4.	$Vol(h)=[(22(D^2)/280000][(H_t-H_{st})^Q-(H_t-h)^Q]/[Q(H_t-H_b)^{Q-1}]$				
Q	2.121	2.0907	2.3741	1.9512	2.2081
N	759	897	1460	5979	1582
R^2	0.9956	0.9918	0.9863	0.9949	0.9921
S	0.0266	0.0131	0.0294	0.0098	0.0298
Total Volume based on Form Factor					
5.	$Vol(ub) = [22(D^2)/280000]*a(H_t)$				
a	0.5051	0.5134	0.4679	0.5493	0.4937
N	69	897	1460	530	1582
R^2	0.9936	0.9863	0.9803	0.9919	0.9905
S	0.0354	0.0203	0.0425	0.0128	0.0430

Table 4 (Ctd). Stem volume and profile models tested by four sets of sample data for *Pinus pinaster*. The Combined data set is the Early plus the Ritson set. Nomenclature is as previously defined with a, b and e constants.

	S.L.Poole	Early	Ritson	Stem Analysis	Combined
Volume Ratio based on Diameter					
6. $v/V = 1+b(d^c/D^e)$ (Burkhart 1977).					
b	-0.5603	-0.5641	-0.3820	-0.5184	-0.5367
c	3.8749	4.0048	2.3489	4.1855	2.4183
e	3.7310	3.8618	2.1724	4.0080	2.3312
N	759	897	1460	5979	1582
R^2	0.9754	0.9624	0.9253	0.9696	0.8972
S	0.0448	0.0649	0.0949	0.0479	0.1058
Volume Ratio based on Heights					
7. $v/V = 1+b(H_t-h)^c/(H_t)^e$ (Cao et al 1980).					
b	-1.1686	-1.0994	-0.8971	-0.6919	-0.7903
c	1.9702	2.0334	2.4054	1.9721	2.1521
e	2.0209	2.0631	2.3676	1.8427	2.0281
N	759	897	1460	5979	1582
R^2	0.9977	0.9981	0.9916	0.9963	0.9917
S	0.0137	0.0146	0.0318	0.0166	0.0301
Volume Ratio based on Heights and Diameter					
8. $v/V = a+b(d/D)+c((H_t-h)/H_t)+e((H_t-h)/H_t)^2$					
a	1.0025	1.0053	0.9980	0.9940	1.0059
b	-0.0639	-0.0407	-0.1769	-0.0388	-0.1222
c	0.1009	0.0951	0.4989	0.0965	0.2837
e	-1.040	-1.0715	-1.2736	-1.0671	-1.1482
N	759	897	1460	5979	1582
R^2	0.9978	0.9981	0.9923	0.9959	0.9908
S	0.0135	0.0144	0.0305	0.0177	0.0316
9. $v/V = 1-b(d/D)^c((H_t-h)/H_t)^e$ (Newberry et al 1989).					
b	0.9938	0.9981	0.9636	2.2147	0.9811
c	0.1654	0.3616	0.1821	0.4429	0.1164
e	1.8901	1.8601	2.2418	1.5227	2.0485
N	759	897	1460	5979	1582
R^2	0.9977	0.9983	0.9918	0.9951	0.9901
S	0.0137	0.0136	0.0315	0.0193	0.0327

height ratio. Newberry and Burkhart (1985) investigating the Ormerod formula with respect to loblolly pine agreed in general with these observations, but found that taper did not appear to decrease with increasing site quality. They confirmed that both taper and form parameters decrease as the combined crown-site index term decreases. This they suggest, results in large crowned, good site trees distributing their growth over the total bole providing trees with a conical bole (ie Ritson data). Small crowned, poor site trees tend to concentrate their annual growth in the area of the live crown, thereby, producing a more cylindrical, less conical tree. This is the case with the closely spaced SLP plantation trees which have relatively small crowns in comparison to the open grown farm planting measured by Ritson.

Table 5. Bias of various models for taper and merchantable volume from the actual values, for the stem analysis data set, age 12-30 years. The diameter deviations (1,2,10) are expressed in cm and the height and volume deviations are expressed as percentage of the actual value. Numbers in brackets refer to the model in Table 4.

h/H	Koz (1)	OrmD (2)	SegD (10)	OrmH (3)	Ormv (4)	Burk (6)	Cao (7)	Par (8)	Newb (9)	Segv (11)
0.0	0.2	0.2	-0.2	65.1	-5.5	-0.1	-17.3	7.0	5.3	-4.5
0.1	-0.2	-0.1	-0.2	-7.9	-1.9	-28.4	-11.5	2.5	1.4	-1.1
0.2	-0.1	0.1	0.1	-1.7	-1.1	-8.2	-4.4	-1.0	-0.7	1.0
0.3	0.0	0.3	0.1	3.3	-1.5	-0.2	-3.4	-1.8	-1.2	1.0
0.4	0.1	0.5	-0.1	2.9	-0.9	1.2	-1.9	-1.2	-0.8	0.5
0.5	0.3	0.8	0.0	4.9	-0.3	3.7	-1.0	-0.5	-0.2	0.6
0.6	0.4	1.0	0.2	4.3	0.0	3.8	-0.1	0.2	0.2	0.5
0.7	0.2	0.7	0.1	0.8	0.3	1.8	0.2	0.7	0.4	0.4
0.8	-0.7	-0.1	-0.1	-4.2	-0.3	-0.5	0.0	0.4	0.0	-0.3
0.9	-2.2	-1.6	0.0	-6.5	-1.4	-1.7	-0.8	-0.5	-0.9	-0.9
1.0	-0.7	-0.5	-0.0	-0.8	-2.4	-0.5	-0.5	-0.2	-0.5	-1.1
ALL	-0.2	0.0	0.0	3.5	-1.3	-3.0	-3.4	0.3	0.1	-0.2

Measurement system - A main objective in measurement of sample trees in the past was volume to 7.5 cm diameter under bark, the theoretical merchantable limit. This value is discarded here as a standard as it has little relation in utilisation practice for older trees. Sample data in this study was related to total tree volume under bark. Many sample trees measured to a merchantable limit had insufficient measurements higher up the stem to provide a realistic estimate of total volume. For the mature South Lane Poole sample which were measured to height 10.5 m a 'realistic' total volume was obtained by using the Kozac model to estimate heights at 13.5, 16.5 and 19.5 m. These sectional heights are 44, 57, 68 and 82 percent of the mean height of the sample data. Volumes to the measured stem height of 10.5 m plus a conical top log were 97.3, 94.4 and 92.7 respectively, of empirical volumes to 13.5, 16.5 and 19.5 m height, considering the top logs of each extrapolation to be conical. The mean form factors to the four top log limits were 0.471, 0.485, 0.499 and 0.509, respectively.

Stems tend to conform to a neiloid frustum at the base, to a paraboloid frustum in the mid section and to a cone for the top log. The form factor for the total stem is of the order of 0.45 to 0.6. Newberry and Burkhart (1985) found a value of 0.6 to be indicative of limiting tree form in loblolly pine, noting that this is not very different to the value of 0.5 for the quadratic paraboloid proposed by Gray (1956). In the stem analysis in which sectional heights were 1.5 m and measurement coverage from ground level to the tree tip was excellent, the form factor was 0.55. With lower percentage heights of the stem measured and increasing relative length (compared to total height) of the top log to be calculated as a cone, the tree form factor tended to approach 0.33.

The process of determining empirical total volume by using a stem taper model to determine upper stem diameters for all other trees with insufficient stem measurement was not used for further data sample sets as the volume obtained could be varied by the length of section interval chosen and the percentage of stem height to which diameters were estimated.

For the current sample data only those trees with stem height measurements up to and above 80 percent of tree height were used. A measurement system similar to the percentile method where stem diameters are measured at percentile heights up the stem (each 10 percent) is required for accurate estimates of total volume.

Table 6. Descriptive Statistics for deviations of predicted from actual values for stem analysis data fitted to other half of data set. Taper data (1,2,10) are in mm while other data are expressed as deviations percent of actual value.

Variable		N	Mean	Median	StDev	SEMean	Min	Max
Koz	(1)	3491	-2.65	-0.15	10.29	0.17	-38.8	31.0
Ormd	(2)	3491	0.83	0.98	10.12	0.17	-34.1	36.0
SegD	(10)	3491	0.02	0.00	6.81	0.11	-35.8	46.6
OrmSH	(3)	3491	3.51	0.00	36.54	0.61	-271.2	66.2
Ormv	(4)	3491	-1.33	-1.07	4.39	0.07	-19.8	12.6
Burkv	(6)	3491	-3.07	-0.64	32.26	0.54	-310.8	99.3
Caov	(7)	3491	-3.42	-1.14	7.71	0.13	-70.5	15.9
Parv	(8)	3491	0.33	0.11	3.82	0.06	-13.2	21.4
Newbv	(9)	3491	0.10	-0.24	3.92	0.06	-14.0	38.4
Segmv	(11)	3491	-0.27	-0.12	4.40	0.07	-19.3	13.1

Test of Models

As a test of the practicability of the models the Stem Analysis data set and the South Lane Poole, Early and Ritson data sets combined were each separated into two equal parts. Equations (Table 4) constructed from the first half of the data were applied to the second half of data to examine the deviation of estimated values from the actual values (Tables 5, 6 and 7).

With the exception of the Ormerod stem height (3) and Burkhart (6) and CaO (7) estimations, the deviations for each class and for the total sample are acceptable. If relative stem heights of 0.2 and greater are used the later two models are acceptable. Max and Burkhart (1976) found for two loblolly pine data sets that there was little difference between alternative models for predictive diameters at relative heights near one. Differences among various models studied were greater when predicted diameters were for lower relative heights, particularly for relative heights less than 0.20.

The standard error of bias for the Ormerod volume formula varied little from the Parabolic (8) and Newberry (9) volume ratio formula which were the best from the point of minimum overall deviation and standard deviation of the differences (Table 6).

The Parabolic volume ratio equation does not require non-linear regression for derivation and may be most suitable in some circumstances.

Table 7. Descriptive statistics for validation of taper and volume models in the Combined data set (1, 2, 3). Equations were calculated for one half of the data and validated against the other half. The taper models are expressed in mm while the volume models are expressed as percent deviation from the actual value.

Variable	N	Mean	Median	StDev	SEMean	Min	Max
DKoz	1534	-0.21	0.00	1.69	0.04	-8.2	6.9
DOrmd	1534	0.11	0.00	1.70	0.04	-8.1	7.0
DdSeg	1534	-0.43	0.00	1.73	0.04	-9.4	4.5
DOrmSH	1534	0.21	0.00	1.17	0.03	-5.1	5.4
%DOrmv	1534	1.59	0.97	9.33	0.23	-34.7	35.5
%DvBurk	1534	0.01	2.99	155.11	3.96	-2309.2	1542.7
%DvCao	1534	-5.46	-0.11	25.25	0.64	-250.2	67.2
%DvPar	1534	2.15	-0.51	16.65	0.42	-123.7	134.6
%DvNewb	1534	-0.08	0.00	13.20	0.33	-154.3	89.4
%DvSeg	1534	0.43	3.39	21.10	0.53	-106.2	45.9

The practicability of using a common value, for the volume formula for the extreme SLP data and Ritson data, is depicted in the combined data set (Table 7). Percentage deviations within diameter classes and totals are within the limits of commercial estimation for the Ormerod (4), Cao (7) and Parabolic (8) and Newberry (9) models, particularly for relative stem heights greater than 0.2. The total deviation involved using the previous adopted 0.49 value in the Ormerod formula for the remaining data sets was -5, -3, and -15 percent for the South Lane Poole, Early and Ritson data sets, respectively. Although a combined alpha value is practical it would be preferable to use alpha values of 0.54 for plantation material and 0.69 for trees planted on farms, if an estimate for the particular sample set was unavailable.

The Volume Formula (4) was equivalent to the Form Factor procedure for determining total volume and hence is satisfactory for all purposes. The alpha values for volume were not always identical to those calculated for merchantable limits to 'd' and 'h' (Table 2). For the SLP data set the alpha values for total and merchantable volume were identical at 0.56. The alpha values calculated for 'd' and 'h' by the Ormerod formulae were slightly less at 0.51. Mean bias for the 0.56, 0.51 and 0.49 alpha values resulted in 2, 3.5 and 4.5 percent overestimate of SLP merchantable values. For practical use these differences are unimportant. Martin (1984) and Cao et al (1980) observed that volumes obtained from integrating the Ormerod diameter model (2), presumably using alpha values calculated from this diameter taper model, did not provide good estimates of merchantable volume.

Models fitted on the basis of diameter limits, for both the Ormerod models (2, 3) and volume ratio models (6, 7) did not fit as well as the models fitted to height limits (Table 2). This was also recorded for volume ratios by Byrne and Reed (1986).

Segmented Models

Since the stem profile varies within a tree (neiloid, parabola, cone) accurate taper and volume models using a single function to depict the whole stem cannot be completely satisfactory over the whole profile. Demaerschalk and Kozac (1977) found that within each species the overall shape of the trees is very much the same for all size classes. Within each group the inflection point (the point where the tree form changes from neiloid to paraboloid) seems to be at a more or less constant relative height regardless of tree size. They fitted two functions to a taper model and found the system to be remarkably precise and accurate. Above breast height standard errors of estimate and biases were negligible for all groups. There was still a slight underestimation below breast height in very large trees. For two loblolly pine data sets, Max and Burkhart (1976) found segmented polynomial models with estimated join points provided an improved description of tree taper when compared to a single quadratic taper model used throughout the stem length. There was little difference between different models for predictive diameters at relative heights near one.

Cao, Burkhart and Max (1980), using loblolly pine data, compared a range of taper, volume ratio and segmented models to determine models that predict most accurately and precisely merchantable volumes to any desired top diameter limit and to any specified height or proportion of total height. The Max and Burkhart segmented model ranked first in ability to estimate tree diameters at various points on the bole. The models constructed for the specific silviculture. The use of the combined model (Table 7), however, provided bias which would normally be within the limits of general operational requirements. The standard errors for models 4, 7, 8 and 9 in the combined data were 0.2 percentage bias from the actual value.

Working Models

For practical use in application to either plantation trees or trees in farm plantings the following formula are offered for general use.

Farm Log formula
$$Vol = 0.0000699(D^{1.866})(H_t^{0.926})1.00435$$
Farm constant form Factor formula
$$Vol = 0.00003733(D^2)H_t$$
Farm Ormerod Full volume formula
$$Vol = (D^2 22/280000)(H_t^{2.4163})/(2.4163((H_t-1.3)^{1.4163})$$

Plantation Log formula
$$Vol = 0.0000327(D^{1.863})(H_t^{1.226})1.00168$$
Plantation constant form Factor formula
$$Vol = 0.0000423(D^2)H_t$$
Plantation Ormerod Full volume formula
$$Vol = (D^2 22/280000)(H_t^{2.0})/(2.00((H_t-1.3)^{1.00})$$

A correction factor (Sprugel 1983) applied to the logarithmic models slightly improved estimation and is included in the above equations.

Table 8. Parameters for segmented models for taper and merchantable volume for the Farm Trees and the Plantation Trees. Parameters are for equations 10 and 11.

$(z-a_1)$	$(z-a_2)$	c	b_1	b_2	b_3
		Farm Taper (a_1= 0.08, a_2 = 0.5)			
-	-	0.035	-17.48		
+	-	0.031	-12.07	59.89	
+	+	0.031	-9.42	61.56	-33.49
		Plantation Taper (a_1 = 0.08, a_2 = 0.8)			
-	-	1.290	0.99		
+	-	0.840	0.43	-1.32	
+	+	0.330	-1.84	3.05	17.32
		Farm Merch Volume (a_1 = 0.08, a_2 = 0.5)			
-	-	0.00048	1.063	-1.361	-13.71
+	-	0.00029	1.199	-3.23	-0.93
+	+	0.00015	1.054	-2.91	-0.114
		Plantation Merch Volume (a_1 = 0.08, a_2 = 0.8)			
-	-	0.000067	2.87	-2.66	-355.33
+	-	0.000011	-4.32	12.01	-22.89
+	+	0.0000035	-11.7	40.5	0.51

The log formula for full volume is considered preferable, generally having both low overall deviations from the actual and low standard deviations for the differences. For taper and merchantable volume it is considered that the Ormerod diameter and volume formula are suitable. The segmented models were slightly superior (Tables 6, 7 and 8) but are difficult to apply and should not be contemplated unless a major analysis is required.

Parameters for equations 10 and 11 to allow estimates of taper and merchantable volume for Farm and Plantation trees are contained in Table 8.

Table 9. Fit of standard formulae for plantation and farm trees back onto the original data set to indicate the suitability of different models. Volumes are expressed as deviation percent of estimated from actual. Taper is expressed as mm deviation of estimated from actual diameters.

Variable	N	Mean	StDev	SEMean	N	Mean	StDev	SEMean
	Plantation Full Volume				Farm Full Volume			
%DevCFF	760	-0.504	6.077	0.220	145	3.369	10.449	0.868
%DevLog	760	0.934	5.657	0.205	145	-0.335	9.197	0.764
%DevOrm	760	-1.337	6.609	0.240	145	-0.380	9.653	0.802
	Plantation Taper				Farm Taper			
DSegD	7563	-0.139	0.898	0.010	1437	-0.057	1.589	0.041
DKozD	7563	2.379	1.791	0.020	1070	-0.065	1.560	0.041
DOrmD	7563	0.034	1.060	0.012	1437	0.169	1.519	0.040
	Plantation Merch volume				Farm Merch Volume			
%DSegV	7563	-0.603	4.997	0.057	1437	4.155	9.154	0.241
%DOrmV	7563	-1.681	4.934	0.056	1437	-1.574	7.950	0.210

It should be noted that most pine established in plantations after 1972 was raised from seed orchard stock and grown under a silviculture regime which favoured lower establishment numbers and early commercial thinning to maximise diameter growth of the final crop stems and reduce the effect of stand density on soil moisture depletion. A suitable sample set was not available for these populations which are expected to adopt a stem profile intermediate between the old plantation growth and farm data sets.

Literature Cited

Bruce, D. Curtis, R. O. and Vancoevering, C. 1968. Development of a System of Taper and Volume Tables for Red Alder. For. Sci., 14:339-350.

Byrne, J. C. and Reed, D. D. 1986. Complex Compatible Taper and Volume Estimation Systems for Red and Loblolly Pine. For. Sci., 32:423-443

Cao, Q. V. Burkhart, H. E. and Max T. A. 1980. Evaluation of Two Methods for cubic-volume Prediction of Loblolly Pine to any Merchantable Limit. For. Sci., 26:71-80.

Demaerschalk, J. P. and Kozac, A. 1977. The whole bole system: a conditioned dual-equation system for precise prediction of tree profiles. Canad. J. For. Res. 7:489-497.

Goulding, C. J. and Murray, J. C. 1976. Polynomial taper equations that are compatible with tree volume equations. N. Z. J. For. Sci. 5:313-322.

Gray, H. R. 1956. The form and taper of forest-tree stems. Univ. Imp. for. Inst. Pap. No. 32 pp. 1-74.

Kozac, A. Munro, D. D. and Smith, J. H. G. 1969. Taper functions and their application in forest Inventory. For. Chron., 45:278-283.

Larson, P. R. 1963. Stem form development of forest trees. For. Sci. Monogr. No. 5.

Max, T. A. and Burkhart, H. E. 1976. Segmented polynomial regression applied to taper equations. For. Sci., 22:283-289.

Martin, A. J. 1981. Taper and volume equations for selected Appalachian hardwood species. USDA Forest Serv. Res. Pap. NE-490, 22p.

Martin , A. J. 1984. Testing volume table accuracy with water displacement techniques. Forest Sci. 30:41-50.

Newberry J. D. and Burkhart H. E. 1985. Variable-form stem profile models for loblolly pine. Canad. J. For. Res. 16:109-114.

Newberry J. D., Burkhart H. E and Amateis R. L. 1989. Individual tree merchantable volume ratios based on geometric solids. Canad. J. For. Res. 19:679-683.

Ormerod, D. W. 1973. A simple bole model. For. Chron., 49:136-138.

Reed, D. D. and Byrne, J. C. 1985. A Simple, Variable form Volume Estimation System. For. Chron., 61:87-90.

Sprugel, D. G. 1983. Correcting for bias in log-transformed allometric equations. Ecology 64:209-210.