FOREST MATHEMATICS

TRAINING MANUAL

FORESTS DEPARTMENT
OF
WESTERN AUSTRALIA

FOREST MATHEMATICS

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CHAPTER 1.

ADDITION AND SUBTRACTION OF DECIMALS AND MONEY

Simple addition and subtraction makes up a major part of every mathematical computation. It is assumed that you know how to add and subtract whole numbers. Some difficulty arises with addition and subtraction of decimals and fractions. This section will deal with decimals, while fractions will be left to a later stage (Chaper 5).

Addition of Decimals

Example 1 : Add 20. 356 to 27. 5358.

These two figures have a different number of decimal places after the decimal point. That is there is a different number of digits on the right hand side of the decimal point. To add these two figures it must be assumed that there are an equal number of digits to the right of the decimal point so a nought is added to give 20. 3560.

```
Then add          20. 3560
                + 27. 5358
          Ans.    47. 8918
```

Example 2 :

Add 27. 31; 4. 3 and 130. 515

Add the appropriate number of noughts to give:

```
           27. 310
            4. 300
          130. 515
Ans.      162. 125
```

When you have got used to the idea the addition of noughts is not necessary. Just ensure that the decimal points of the two or more numbers to be added are placed one under the other.

Example 3 :

Add 25. 8538 and 6. 35

```
            25. 8538
          +  6. 35
Ans.        32. 2038
```

The same rule applies when there is no whole number in front of the decimal point.

Example 4 :

Add 237. 23 to 0. 0613

```
           237. 23
         +   0. 0613
Ans.       237. 2913
```

Addition of Decimal Currency

Problems may arise when adding amounts ending in a decimal point of a cent.

Example 5 :

Add 25 dollars and 23.2 cents and 56 dollars and 18.4 cents.

One approach would be to convert each amount to cents then reconvert to dollars following addition.

```
 2523.2
 5618.4
 8141.6
```

Ans. : 81 dollars and 41.6 cents.

Subtraction

When a small number is to be deducted from a large one no difficulty is encountered. Problems may arise when a large number is subtracted from a smaller one.

Example 6 :

Subtract 4535 from 257.

The easiest method is to reverse the position of the figures and set them down as

```
- 4535        instead of      257
   257                     - 4535
  4278
```

and then subtract 257 from 4535 and put a minus sign in front of the answer.

Ans. : - 4278

Subtraction of Decimals

To subtract one number from another with different numbers of places after the decimal point treat the same as for addition by adding the required number of noughts.

Example 7 :

Subtract 517.37 from 724.4

```
        724.40
      - 517.37
Ans.    207.03
```

Where the subtracted number is larger than the other, do as in Example 6, remembering to keep the decimal place of one figure below the other.

Example 8 :

Subtract 21.87 from 0.93

```
      - 21.87
         0.93
Ans.  - 20.94
```

Problems :

1. Add 25.635, 8.45, 3.7885 and 4.131.
2. Add 6.0031, 0.06, 261.4.
3. Add 25 dollars 30.31 cents, 6 dollars 90.4 cents and 107 dollars 6.2 cents.
4. Subtract 421 from 35.
5. Subtract 241.316 from 721.2.
6. Subtract 36.006 from 0.04.
7. Subtract 0.00063 from 271.2.
8. Subtract 721.361 from 947.472.

CHAPTER 2.

MULTIPLICATION

Multiplication is an expansion of the concepts involved in addition. In multiplying 2 x 3, two is added three times; 2 + 2 + 2 = 6 (or three is added two times; 3 + 3 = 6). This applied to a problem 328 x 3 means 328 + 328 + 328 = 984. This is the way in which a calculating machine does multiplication.

The accepted method for setting out multiplication calculations is as below.

Example 1 :

Multiply 258 by 25.

In fact we multiply 258 first by 5 and then by 20 and add

```
          258
        x  25
         1290
       + 5160
Ans.     6450
```

Example 2 :

Multiply 3875 by 2168

```
             3875
           x 2168
            31000
           232500
           387500
        + 7750000
Ans.      8401000
```

Multiplication of decimal figures is carried out the same way but the number of decimal places (number of digits to the right of the decimal point) have to be taken into account.

Example 3 :

Multiply 6.2 x 4.7

Multiply as if there were no decimal points.

```
   6.2
 x 4.7
  43 4
 248 0
 291 4
```

Since there is one decimal place in each of the two numbers multiplied, we add these and reinsert two decimal places in the answer, counting from the right.

Ans. : 29.14

Example 4 :

Multiply 26.41 by 3.2

```
  26.41 ........ 2 decimal places
x  3.2  ........ 1 decimal place
  52 82
 792 30
 845 12
```

Since there is a total of three decimal places in the numbers multiplied three decimal places must be reinserted in the answer.

Ans. : 84.512

Example 5 :

Multiply 22.004 by 0.006

```
  22.004 ....... 3 decimal places
x   .006 ....... 3 decimal places
  132024
```

Reinsert six decimal places in the answer.

Ans. : 0.132024

Example 6 :

Multiply 289.531 by 8.

```
  289.531
x       8
  2316248
```

Ans. : 2316.248

In multiplying decimal currency the method is similar.

Example 7 :

Multiply $8.56 by 20.8

```
  8.56 ........ 2 decimal places
 20.8  ........ 1 decimal place
 68 48
171200
178048
```

Reinsert 3 decimal places

Ans. : 178.048
or 178 dollars 4.8 cents
or $178.05 to the nearest cent.

Problems :

1. Multiply 241 by 73.
2. Multiply 2006 by 23.
3. Multiply 147 by 102.
4. Multiply 27.43 by 8.2.
5. Multiply 287.004 by 0.6.
6. Multiply 242.3 by 1.003.
7. Multiply 43.0043 by 0.0041.
8. A truck has a capacity of 5.6 cu. yards of gravel and the contractor charges $1.58 to cart one cu. yard. How much will it cost for 45 truck loads?

9. A weedicide solution required 0.056 pints of weedicide per gallon of water. How many pints of weedicide must be put in a 150 gallon tank of water?

10. The cost of running a certain landrover is 10.03 cents per mile. What is the total cost of a trip of 102 miles?

11. A mature stand has an average volume of 82.73 loads per acre and an area of 100.23 acres. What is the total volume in the stand?

```
      100.23
       82.73
    --------
       30069
      70161
     20046
    80184
  ----------
   8292.0279
```

CHAPTER 3.

DIVISION

Division can be thought of as an expansion of the process of multiplication. When 8 is divided by 2 we are saying "How many times can two be taken away from 8." The answer is 4. Similarly to divide 33 by 11 is to ask how many times 11 can be taken away from 33. The answer of course is 3. The process is the reverse of multiplication.

The method of setting out division is as follows.

Divide 288 by 4.

72	
4) 288	4 into 2 won't go
28	4 into 28 = 7
. . 8	Bring down the next figure
8	4 into 8 = 2
.	

Ans. : 72

Example 2 :

Divide 1148 by 41.

28	
41) 1148	41 into 11 won't go but
82	41 into 114 will go twice
328	Multiply 2 x 41 - 82 and
328	subtract from 114 = 32.
. . .	Bring down the next figure
	41 into 328 = 8 times.

Ans. : 28

When a number does not divide exactly into another exactly to give a whole number answer, the amount "left over" can be expressed as a fraction or a decimal.

To express as a fraction.

Example 3 :

Divide 276 by 8.

34	
8) 276	8 into 27 = 3
24	3 x 8 = 24 and subtract from 27.
36	Bring down the next figure
32	8 into 36 = 4 with 4 left over
4	

Ans. : 34 and 4/8 left over.
or $34\frac{1}{2}$.

To express as a decimal.

Example 4 :

Divide 276 by 8.

34. 5	
8) 276	8 into 27 = 3
24	3 x 8 = 24 and subtract from 27 = 3
36	Bring down the next figure
32	8 into 36 = 4, 4 x 8 = 32
40	Subtract from 36 and bring
40	down a 0 and put a decimal
..	place in the answer.
	8 into 40 = 5

Ans. : 34. 5

Example 5 :

Divide 31 by 8.

3. 875	
8) 31	8 into 31 = 3, 3 x 8 = 24 Sub-
24	tract from 31 = 7
70	Bring down a 0 and insert decimal
64	in answer. 8 into 70 = 8, 6 left over
60	Bring down another 0
56	8 into 60 = 7, and four left over
40	Bring dow another 0.
40	8 into 40 = 5 exactly.
..	

The number of decimal places to which the answer is taken will depend upon the accuracy required. For example we may want to record the average growth rate of a tree over a 10 year period to two decimal places (0. 17 inches per annum). Vehicle fuel consumption is usually taken to one decimal place (21. 3 miles per gallon). In calculating the number of pines needed to plant an area we would not want 0. 5 of a pine tree but would take the figure to the nearest tree. Money calculations are usually taken to the nearest cent, not to the nearest dollar.

Division of Decimals :

When dividing figures containing decimals, use the following procedure.

Example 6 :

Divide 28. 5 by 25.

Both figures should have the same number of figures on the right hand side of the decimal point. That is 28. 5 divided by 25. 0. Then drop the decimal points and divide normally.

```
              1.14
     250 ) 285
           250
            350        Bring down 0 and insert decimal
            250        in answer.
            1000
            1000
           ......
     Ans. : 1.14
```

Example 7 :

Divide 321.75 by 7.9

Shift the decimal points over to the right two places.

i.e.

```
               40.72
     790 ) 32175
           3160
             5750      Put in a decimal point
             5530      and bring down the 0
              2200
              1580
               620 etc.
```

Example 8 :

Divide 0.63718 by 0.0031

Shift the decimal point to the right four places.

```
              205.5
     31 ) 6371.8
          62
           171
           155
            168
            155
             13 etc.
```

Always make sure that the decimal point in the answer goes directly above the decimal point in the number being divided.

Decimal Currency :

With decimal currency the process of division is fundamentally the same. Care must be taken when dollars and cents are involved.

The best proceedure is to convert dollars to cents and then proceed with the calculation.

Example 9 :

Divide $816.50 by 81.3.

Convert dollars to cents

= 81650 cents.

Shift the decimal point one place to the right. Then

```
           1004.3
813 ⟌ 816500
      813
        3500
        3252
         2480
         2439
           41
```

Ans. : 1004.3 cents or $10.04 to the nearest cent.

Dividing a large into a smaller figure

This really poses no problem as long as you remember to place the decimal point in the answer immediately above the decimal point of the number being divided.

Example 10 :

Divide 16 by 352

```
          .045
352 ⟌ 16.00
      14 08
       1 920
       1 760
         1600 etc.
```

Ans. : 0.045

Rounding off to a Specified Number of Decimal Places

As mentioned previously in this section we usually decide beforehand upon the number of decimal places to which the answer will be taken. This requires that the figure we obtain for an answer be rounded off. A figure like 26 dollars and 35.3 cents is rounded off to the nearest cent, $26.35. Or 23.77 mpg is rounded off to 23.8 mpg. That is we round off to the nearest figure. A figure containing 0.5 is always rounded off to the next highest figure. For example 27.5 trees becomes 28 trees, or 24.35" d.b.h. becomes 24.4" d.b.h.

Problems :

1. Divide 725 by 25.
2. Divide 8751 by 220 and take the answer to 3 decimal places.
3. Divide 861 by 215 and take the answer to 4 decimal places.
4. Divide 528.78 by 26.8 and take the answer to 3 decimal places.
5. Divide $865.20 by 26.6. Give the answer to the nearest cent.
6. Divide .03792 by 0.00061. Answer to two decimal places.
7. Divide 76 by 129. Answer to 3 decimal places.
8. Divide 230.5 by 851.6. Give answer to 5 decimal places.
9. Round off the following figures :

   5.7757 to 2 decimal points
   17.55 to 1 decimal point
   0.005 to 2 decimal points
   3.46542 to 3 decimal points.

CHAPTER 4.

SQUARES, CUBES AND OTHER POWERS

When 2^2 is written it means 2 x 2 or 2 multiplied by itself two times. This is referred to as "2 squared".

Similarly 2^3 (spoken of as "2 cubed') means 2 x 2 x 2 or 2 multiplied by itself three times.

We can take 2 to any power. For example 2^5 (expressed as "2 to the fifth") will be 2 x 2 x 2 x 2 x 2 = 32.

The square, cube or any other power can also be found for any other number.

Examples :

3^2 (expressed "3 squared")	= 3 x 3 = 9
5^3 (expressed "5 cubed")	= 5 x 5 x 5 = 125
4^4 (expressed "4 to the fourth")	= 4 x 4 x 4 x 4 = 256
2.5^2 (expressed "2.5 squared")	= 2.5 x 2.5 = 6.25
0.4^2 (expressed "0.4 squared")	= 0.4 x 0.4 = 0.16

Example 1 :

Find 1.37^3

Multiply 1.37 by itself 3 times.

```
      1.37
    x 1.37
       959
      5110
     13700
    1.9769
    x 1.37
    138383
    593070
   1976900
  2.708353
```

Problems :

1. Express 3 x 3 x 3 x 3 x 3 x 3 as a power of three.
2. Cube 4.
3. Calculate 10^4.
4. Find 2.31 cubed.
5. Find 0.00310 squared.
6. Find 2.006^3.

CHAPTER 5.

H.C.F. AND L.C.M. USE WITH FRACTIONS

It is here necessary to define a number of terms :

Prime Numbers
Fraction
Numerator
Denominator
Highest Common Factor (HCF)
Lowest Common Multiple (LCM)

Prime Number

A prime number is a whole number which is not exactly divisible by any number (with the exception of 1) but itself to give a whole number answer.

e.g. 1, 2, 3, 5, 7, 11, 13, 17, 23, 29, 31 ...... etc.

Fraction

A fraction is a part of a whole number and is expressed as a numerator over a denominator.

e.g. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ etc.

Fractions can be called either "proper" or "improper". A proper fraction cannot be expressed in any other fractional way but an improper fraction can.

e.g. Proper $\frac{1}{3}$, $\frac{1}{4}$

Improper 8/5, 7/1 (These can be expressed $1\frac{3}{5}$ and $3\frac{1}{2}$ respectively)

Numerator

When using fractions the numerator is the figure on the "top".

e.g. in the fraction $\frac{3}{4}$, 3 = the numerator.

Denominator

When using fractions the denominator appears on the "bottom".

e.g. in the fraction $\frac{3}{4}$, 4 is the denominator.

Highest Common Factor

When two numbers are compared the Highest Common Factor is defined as the largest number that will divide into each number to give a whole number answer.

Lowest Common Multiple

When two numbers are compared the Lowest Common Multiple is the smallest number into which both numbers will divide to give a whole number answer.

There are a number of ways to calculate HCF and LCM.

By Prime Numbers (or Factors)

Example 1 :

First H.C.F. and L.C.M. of 540 and 243

$540 = 2^2 \times 3^3 \times 5$

$243 = 3^5 \; (= 3^3 \times 3^2)$

The factor common is :

$3^3 \; \therefore$ H.C.F. $= 3^3$ or 27

The lowest number into which each will divide is :

$3^5 \times 2^2 \times 5$

$\therefore$ L.C.M. = $3^5 \times 2^2 \times 5$ or 4860

Ans. : L.C.M. = 4860 and H.C.F. = 27

H.C.F. by Repeated Division

Example 2 :

Divide 540 by 243

```
              2
    243 ) 540
        - 486      4
          54 ) 243        Divide 243 by 54
             - 216
               27 ) 54    Divide 54 by 27
```

27 goes exactly so 27 is the H.C.F.

Ans. : 27

L.C.M. Simultaneous Method

Example 3 :

Find prime factors of both simultaneously.

3)	540	243
3)	180	81
3)	60	27
3)	20	9
3)	20	3
2)	20	1
2)	10	1
	5	1

Ans. : $3^5 \times 2^2 \times 5$

Just move the numbers down when a selected divisor will not go into it e.g. 20 is moved down when it cannot be divided by 3.

Example 4 :

Find L.C.M. of 27, 36, 45

3)	27	36	45
3)	9	12	15
3)	3	4	5
2)	1	4	5
2)	1	2	5
	1	1	5

Ans. : $3^3 \times 2^2 \times 5$

Fractions

When addition and subtraction of fractions is necessary L.C.M. is used.

APPENDIX

A. Conversion Factors

	IN ONE	THERE ARE
LENGTH	meter (m)	1×10^{-3} km, 100.0 cm, 1000 mm, 39.37 in.
	kilometer (km)	1.000×10^{3} m
	centimeter (cm)	1.000×10^{-2} m
	inch (in.)	2.540×10^{-2} m
	foot (ft)	0.3048 m
	mile (mi)	1609 m
	micron (μ)	1.000×10^{-6} m
	angstrom (Å)	1.000×10^{-10} m
	light-year	9.464×10^{15} m
AREA	square meter (m^2)	1.000×10^{4} cm^2, 1.550×10^{3} $in.^2$
	square inch ($in.^2$)	6.452×10^{-4} m^2
	square foot (ft^2)	0.0929 m^2
VOLUME	cubic meter (m^3)	1.000×10^{6} cm^3, 35.31 ft^3, 264.2 gallons
	cubic foot (ft^3)	0.02832 m^3
	liter (l)	1.000×10^{3} cm^3, 0.2642 gallon
VELOCITY (SPEED)	meter per second (m/sec)	3.281 ft/sec, 0.3728 mi/min
	mile per minute (mi/min)	88.00 ft/sec, 26.82 m/sec
	mile per hour (mi/hr)	1.467 ft/sec, 0.4470 m/sec
MASS	kilogram (kgm)	1000 gm
	pound mass (lbm)	0.4536 kgm
	atomic mass unit (amu) (physical scale)	1.660×10^{-27} kgm
FORCE	newton (new)	1.000×10^{5} dynes, 7.233 poundal, 0.2247 lbf
	pound force (lbf)	4.448 new, 32.17 poundal
	poundal	0.1383 new, 0.0311 lbf

Example 5 :

Add $\frac{1}{3}$, $\frac{5}{8}$ and $\frac{3}{4}$

$\frac{1}{3} + \frac{5}{8} + \frac{3}{4}$

$\frac{8 + 15 + 18}{24}$

$= \frac{41}{24}$

Ans. : $1\frac{17}{24}$

by quick inspection it is obvious that 24 is the smallest number into which the denominators will divide eveny.

Example 6 :

Subtract $\frac{9}{32}$ from $\frac{3}{10}$

$\frac{3}{10} - \frac{9}{32}$

$\frac{48 - 45}{160}$

Ans. : $\frac{3}{160}$

The lowes number into which both 32 and 10 vill divide is 160.

Problems :

1. Find the H.C.F. and L.C.M. of 35 and 70.
2. Find the H.C.F. and L.C.M. of 280 and 15.
3. Find the H.C.F. and L.C.M. of 1760 and [illegible]00.
4. Find the H.C.F. and L.C.M. of 85, 120 ad 375.
5. Add $\frac{1}{8}$, $\frac{3}{5}$ and $\frac{5}{7}$.
6. Subtract $\frac{5}{8}$ from $\frac{7}{10}$.
7. Add $\frac{7}{25}, \frac{5}{8}, -\frac{3}{10}, \frac{9}{11}$.

CHAPTER 6.

SQUARE AND CUBE ROOT

In the previous two sections we have covered the background work necessary to understand square and cube root. We saw that 2^2 meant 2 x 2 and then that a number such as 270 could be broken down to 3^3 x 2 x 5 (prime factors).

Square Root

What is square root? It is the process of finding a number which, raised to the power of two, would give you the number you already have.

Example 1 :

What is the square root of 4?

The number raised to the power of 2 which would give 4 is 2,

$2^2 = 4$

Then 2 is the square root of 4.

Cube Root

Cube root is the process of finding a number which, raised to the power of three, would give you the number you already have.

Example 2 :

What is the cube root of 8?

The number raised to the power of 3 which would give 8 is 2

$2^3 = 8$

Then 2 is the cube root of 8.

The usual symbols for square and cube root are $\sqrt{}$ and $\sqrt[3]{}$ respectively. Thus if we want the square root of 4 we would put down $\sqrt{4}$ and the cube root of 8 as $\sqrt[3]{8}$.

Example 3 :

Find the square root of 64. ($\sqrt{64}$)

Bringing it down to its prime factors is one method.

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^6 = 2^2 \times 2^2 \times 2^2$

Then $\sqrt{64} = \sqrt{2^2 \times 2^2 \times 2^2}$

$= 2 \times 2 \times 2 = 8$

Ans. : 8

Check : 8 x 8 = 64

Example 4 :

Find the cube root of 64.

As above

$64 = 2^6$

$= 2^3 \times 2^3$

Then $\sqrt[3]{64} = \sqrt[3]{2^3 \times 2^3}$

$= 2 \times 2$

$= 4$

Ans. : = 4

Check : 4 x 4 x 4 = 64

Example 5 :

Find the square root of 3600.

```
2 )   3600
2 )   1800
2 )    900
2 )    450
5 )    225
5 )     45
3 )      9
         3
```

$= 2^4 \times 5^2 \times 3^2$

$\sqrt{3600} = \sqrt{2^4 \times 5^2 \times 3^2}$

$= 2^2 \times 5 \times 3$

∴ Ans. : 60

An alternative way of calculating the square root is known as the SYNTHETIC METHOD. This method is not to be confused with Division.

Example 6 :

Find the square root of 225 using the synthetic method.

```
Position A ______   1 x (5)
Position B __1   ) 02 25
Position C ______   1
Position D  2x(5)   125
                    125
                    ....
```

Steps

1. Group digits in pairs, working outward from the decimal point. (This necessitates the addition of a "0" in front of the 2 to complete the pair)

2. Find the number which squared is less than or equal to the first pair of digits. (In this case 1^2 will go into 2)

3. Proceed in a similar method to division putting the figure 1 in the 3 positions shown. (A, B & C)

4. Subtract 1 from 02 and bring down the next pair of digits.

5. Double the "1" in position B and place it in position D.

6. Now find the number "X" which when multiplied by 2X will be less than or equal to 125. (In this case X = 5 since 5 x 25 = 125)

In this case had we tried X = 6 we would have 6 x 26 = 156 which is too big.

Ans. : 15

Example 7 :

Find the square root of 3600.

```
          60
6   )   3600
        36
60      0000   .... Since 60 x 0 = 0000
        0000
        ....
```

Ans. : 60

Example 8 :

Find $\sqrt{63001}$

```
              2
    2 )   6 30 01
          4
  4(5) ) 230
         225
*50(1)  )  501
           501

  Ans. : 251
```

* Where did the number 50 come from? From the number 45 subtract the last digit (in this case 5) 45-5=40. Double the last digit ((5) x 2 = 10) and add this to the 40 obtained above. For larger numbers repeat this process at each stage.

When dealing with decimal quantities the groups of digits are still paired off working outwards from the decimal point.

Example 9 :

Find the square root of 6.3001

```
              2. 51
    2 )   06. 30 01
          4
  4(5) )  2 30
          2 25
 50(1)  )    501
             501

  Ans. : 2.51
```

Bring down 30 and put in decimal point in answer.

Bring down 01.

Note that if there are 4 decimal places in the figure for which the square root is to be found the answer will have only 2 decimal places. If the square root of a number with less than an even number of decimal places is to be found first add a 0 to make an even number e.g. 35.865 becomes 35.8650

Example 10 :

Find the square root of 125.68351

```
              11.210
        )  1 25. 68 35 10
           1
   2(1)  ) 025
            21
  22(2)   ) 468
            444
 224(1)   ) 2435
            2241
2242( )   ) 19410
```

Ans. : 11.210 (Note this is not the exact square root but it is only necessary to do further calculation if higher accuracy is required.)

Problems

1. Using prime factors find the square root of 1089.
2. Using prime factors find the square root of 202500.
3. Using prime factors find the cube root of 721.
4. Using prime factors find the cube root of 46656.
5. Use the synthetic method to calculate the square root in problems 1 and 2.
6. Find $\sqrt{225}$ by the synthetic method.
7. Find $\sqrt{53361}$.
8. Find $\sqrt{8651}$ to one decimal place.
9. Find $\sqrt{256.835136}$.
10. Find $\sqrt{7.8961}$
11. Find $\sqrt{88.356}$.

CHAPTER 7

RATIO AND PROPORTION

When cooking a cake the housewife adds certain quantities of each ingredient. She must not vary these given quantities unduly or her cake will be a failure.

Likewise a man mixing concrete must add a specified quantity of each ingredient to get a desired product. If he does not add enough cement the batch will be too weak. If he does not add enough aggregate and sand he will be wasting costly cement.

In either of the above cases the principle of ratio and proportion is involved. In the latter case the mixture could be :

4 buckets of blue metal

2 buckets of sand

1 bucket of cement

The ratio of the ingredients is said to be 4 is to 2 is to 1, or 4:2:1. The proportions are 4 units of metal, 2 units of sand and 1 unit of cement.

Many problems arise daily in which the principle of ratio and proportion are involved.

Ex. 1 :

Concrete is made up from metal, sand and cement mixed in the following ratio :

3:2:1

How much sand and metal will be necessary if 3 buckets of cement are used? Call metal X and sand Y.

When a bucket of cement is used 2 of sand and 3 of metal are required.

$\therefore$ when 3 of cement are used it will be necessary to multiply other quantities by 3.

3:2:1

$3 \times 3 : 2 \times 3 : 1 \times 3 = 9:6:3$

Ans. : 9 buckets of metal and 6 of sand.

Ex. 2 :

Using the data in Ex. 1 how much cement and metal would be required to mix with $3\frac{1}{2}$ buckets of sand?

Ratio : 3:2:1

$X:3\frac{1}{2}:Y$

Where X and Y = metal and cement.

From the ratio it is obvious that twice as much sand as cement is used.

$\therefore$ Amount of cement = $1\frac{3}{4}$ buckets.

Half as much metal again as sand is used.

$= 3\frac{1}{2}$ and $1\frac{3}{4} = 5\frac{1}{4}$

$\therefore$ ratio $= 5\frac{1}{4} : 3\frac{1}{2} : 1\frac{3}{4}$

Ans. : $5\frac{1}{4}$ buckets metal, $1\frac{3}{4}$ buckets cement.

CHECK

Put into improper fraction form and put over a common multiple

$\frac{21}{4} : \frac{7}{2} : \frac{7}{4}$

$\frac{21 : 14 : 7}{4}$

the ratio 21 : 14 : 7 when divided by 7 = 3 : 2 : 1

$\therefore$ the ingredients are in the correct proportions.

Problems quite often do not have three variables involved. Most problems involve only two variable quantities.

Ex. 3 :

To make a quantity of drink, 1 gallon of juice extract is mixed with 5 gallons of water. How much juice extract will be mixed with $12\frac{1}{2}$ gallons of water?

Ratio 1 : 5 then x : $12\frac{1}{2}$

Divide 5 into $12\frac{1}{2}$ to see how much juice extract must be added.

$$5 \overline{)\,12.5} = 2.5$$

$$= 2\frac{1}{2}$$

Ans. : $2\frac{1}{2}$ gallons of juice extract.

This can be set out in the following way :

1 gallon to 5 gallons

x gallon to $12\frac{1}{2}$ gallons

then multiply 1 by $12\frac{1}{2}$ and divide by 5

$$x = \frac{12\frac{1}{2} \times 1}{5}$$

$$= \frac{\overset{5}{\cancel{25}}}{2} \times \frac{1}{\cancel{5}}$$

$$= 2\frac{1}{2} \text{ gallons}$$

Ans. : $2\frac{1}{2}$ gallons

Ex. 4 :

5 sheep cost $25. How much will 63 cost?

5 : 25 63 : x

or 5 sheep = $25

and 63 sheep = $ (x)

Cross multiply (i. e. 5 x (x) and 63 x 25)

then 5 x x = 63 x 25

Divide both sides by 5

then $x = \frac{63 \times 25}{5}$

Ans. : x = $315

It will be sometimes necessary to split a quantity up into its original proportions.

Ex. 5 :

Pine timber is supplied to the Metropolitan area from three locations

Conversion Factors for English and Metric Units

To convert column 1 into column 2, multiply by	Column 1	Column 2	To convert column 2 into column 1, multiply by
	Length		
0.621	kilometer, km	mile, mi	1.609
1.094	meter, m	yard, yd	0.914
0.394	centimeter, cm	inch, in	2.54
	Area		
0.386	kilometer², km²	mile², mi²	2.590
247.1	kilometer², km²	acre, acre	0.00405
2.471	hectare, ha	acre, acre	0.405
	Volume		
0.00973	meter³, m³	acre-inch	102.8
3.532	hectoliter, hl	cubic foot, ft³	0.2832
2.838	hectoliter, hl	bushel, bu	0.352
0.0284	liter	bushel, bu	35.24
1.057	liter	quart (liquid), qt	0.946
	Mass		
1.102	ton (metric)	ton (English)	0.9072
2.205	quintal, q	hundredweight, cwt (short)	0.454
2.205	kilogram, kg	pound, lb	0.454
0.035	gram, g	ounce (avdp), oz	28.35
	Pressure		
14.50	bar	lb/inch², psi	0.06895
0.9869	bar	atmosphere,* atm	1.013
0.9678	kg(weight)/cm²	atmosphere,* atm	1.033
14.22	kg(weight)/cm²	lb/inch², psi	0.07031
14.70	atmosphere,* atm	lb/inch², psi	0.06805
	Yield or Rate		
0.446	ton (metric)/hectare	ton (English)/acre	2.240
0.891	kg/ha	lb/acre	1.12
0.891	quintal/hectare	hundredweight/acre	1.12
1.15	hectoliter/ha, hl/ha	bu/acre	0.87
	Temperature		
$\left(\frac{9}{5}\,°C\right) + 32$	Celsius, C	Fahrenheit, F	$\frac{5}{9}\,(°F - 32)$
	−17.8°	0°	
	0°	32°	
	20°	68°	
	100°	212°	

* The size of an "atmosphere" may be specified in either metric or English units.

in the ratio $\frac{3}{4} : 1 : 2$. What quantity is supplied from each source to make up 6000 loads?

Change the ratio to whole numbers by multiplying through by 4.

$4 \times \frac{3}{4} : 4 \times 1 : 4 \times 2 \quad = 3 : 4 : 8$

Total $3 + 4 + 8 = 6000$

Then $\frac{3}{3+4+8}$ of $6000 = \frac{3}{15} \times 6000 \quad = 1200$ loads

$\frac{4}{15}$ of $6000 = 1600$ loads

$\frac{8}{15}$ of $6000 = 3200$ loads

Ans. : 1200 loads, 1600 loads and 3200 loads.

CHECK

Add all together.

$1200 + 1600 + 3200 = 6000$ loads.

PROBLEMS

1. Concrete is made with 5 parts metal, 3 parts sand and 1 part cement. How much sand and metal are required for a batch of concrete using $3\frac{1}{2}$ buckets of cement?
2. Petrol is mixed with oil in the ratio $8 : \frac{1}{2}$. How much oil must be added to 24 gallons of petrol?
3. Logs put through a mill produce waste and sawn timber in the ratio 2 : 1. What volume of timber is produced from 60 cubic feet?
4. A load of 600 super feet of timber is sent to a building site. Some is lost due to poor tieing down, some is damaged too badly to use and the remainder gets to its destination safely. The ratio of loss to damaged to delivered safely is 3 : 4 : 33. How much timber is lost?
5. Stacked, sawn timber at a mill is either sold, discarded due to warping, or stolen in the ratio 20 : 4 : 1. Some 20 loads have been discarded. How much is sold and how much is stolen?
6. A price of $375 is paid for 2800 super feet of timber. How much would be paid for 6000 super feet?

CHAPTER 8

AVERAGES

Averages are used in all sciences and no less in forestry. In a pine plantation, the average height or diameter of a stand is used as a measure of growth.

The average capacity of a mill to process timber is a necessary piece of data before allowing for an annual cutting coupe. The mill may consume a different amount of log timber on each day of its operation, but over a year's operation an average can be struck.

An average is arrived at by adding up a set of numbers and then dividing the product by the number of entries contributing to the total.

Ex. 1 :

Find the average of 10, 4, 6, 3 and 7.

There are 5 numbers to be averaged.

```
        10
         4
         6
         3
         7
        ---
Total   30
```

Average $= \frac{30}{5} = 6$

Ans. : Average = 6

The answer is often not a whole number as in Ex. 1 in which case the result can be expressed as a fraction or a decimal quantity.

Ex. 2 :

Average 8, 4, 3, 7, 6 and 3.

```
        8
        4
        3
        7
        6
        3
        --
Total   31
```

Average $= \frac{31}{6} =$ 5.1/6 or 5.167 (approx.)

Ans. : Average = 5.1/6 or 5.167 (approx.)

Ex. 3 :

Average 20.3, 18.6, 5.8, 3.65, 7.12

20.3 + 18.6 + 5.8 + 3.65 + 7.12

Total 55.47

Average $= \frac{55.47}{5} = 11.094$

Ans. : Average = 11.094

Ex. 4 :

The height of a number of trees are measured and found to be 46.5 ft., 53.0 ft., 61.5 ft., 38.5 ft., 55.0 ft. Find the average height.

46.5 + 53.0 + 61.5 + 38.5 + 55.0

Total 254.5

$$\text{Average} = \frac{254.5}{5} = 50.9 \text{ ft.}$$

Ans. : 50.9 ft.

PROBLEMS

1. Find the average of the following figures :
   27.5, 86.1, 306.1, 28.4, 17.1.
2. Find the average of the following figures :
   0.3576, 8.638, 0.7535.
3. The total height of 25 trees is 1325 feet. What is the average height?

CHAPTER 9

CONVERSION OF MEASUREMENT UNITS

This section is designed to familiarise the student with processes used to convert from one unit of measurement to another.

Reference should be made to the tables set out in the appendix (1-4) while following the worked problems through. It is not intended that all the values set out in the conversion tables be memorised. However, the student should be fully familiar with the commonly used units.

Some problems have been set on the conversion of British to Metric units. The intention here is to make the student aware of the fact that figures read in publications need not remain as unknown quantities. For instance : cubic metres are often used in overseas literature rather than cubic feet or super feet. It is therefore necessary to know how to convert cubic metres to super feet.

This section will be dealt with under the following headings :

1. Conversion of linear units.
2. Conversion of square units.
3. Conversion of cubic and liquid units.
4. Conversion of weight units.
5. Conversion of mixed units.

1. LINEAR MEASURE CONVERSIONS

Ex. 1 :

Convert 1 chain 8 yds. 2 ft. 5 ins. to inches.

1 chain = 66 ft., 1 yard = 3 ft., 1 foot = 12 ins.

1 chain x 66 = 66 ft.

8 yds. x 3 = 24 ft.

\+ 2 ft. = 92 ft. x 12 = 1104 inches

\+ 5 inches

1109 inches

Ans. : 1109 inches.

Ex. 2 :

Convert 1 mile 8 chains 15 yds. to feet.

1 mile		=	5,280 feet
8 chains	= 8 x 66 ft.	=	528 feet
15 yds.	= 15 x 3 ft.	=	45 feet
	Total		5,843 feet.

Ans. : 5843 feet.

Ex. 3 :

Find the number of chains and links in 1,795 feet.

1 chain = 66 feet

X chain = 1795 feet

$X = \frac{1795}{66}$ chn. (by cross multiplication and division of both sides by 66)

```
         27.13/66
66 ) 1795
     132
      475
      462
       13
```

At this stage 13 can be multiplied by 1.515 to give links. (There are 1.515 links per foot.)

1.515 x 13 = 19.695 links

= 20 links when taken to the nearest link.

Ans. : 27 chains 20 links approx.

However, go back to the division above again : If the division is continued the answer is : 27.19696. Take away the 27 chains and the figures remaining are very nearly the same as those resulting from multiplying 13 x 1.515. Multiply the 0.19696 by 100 to bring it to links and the answer is : 19.696 links. This is an easier method of calculating the number of links than multiplying the number of feet by 1.515.

Ex. 4 :

Convert 3825 feet to chains, links and decimal parts of a link. (Rounded off to the first decimal place only.)

```
         57.9545
66 ) 3825
     330
      525
      462
       630
       594
        360
        330
         300
         264
          360
          330
           30
```

Ans. : 57 chains 95.5 links.

Ex. 5 :

Convert 16,583 links to chains and feet. (To nearest foot.)

1 chain = 100 links

x = 16,583 links

Step 1 : $x = \frac{16583}{100}$ chains

x = 165 chains and 0.83 of a chain

Step 2 : 1 chain = 66 ft.

0.83 chains = Y feet

Y = 0.83 x 66 = 54.78 feet

= 55 feet approx.

Ans. : 165 chains 55 feet.

Ex. 6 :

Convert 8 links to feet, inches and decimal parts of an inch.

1 link = 7.92 inches

8 links = X

$\therefore$ X = 8 x 7.92 ins.
= 63.36 ins.
= 5 feet 3.36 ins.

Ans. : 5 feet 3.36 ins.

Ex. 7 :

How many yards are there in 1500 metres?

1000 metres = 1093.611 yards

1500 = X

$X = 1093.611 \times \frac{1500}{1000}$ yards

Ans. : 1640.418 yards.

Ex. 8 :

Convert 7 inches to centimetres.

1 inch = 2.540 cms.

7 " = X

X = 7x 2.540 cms.

Ans. : 17.780 cms.

Ex. 9 :

Convert 58 centimetres to inches.

2.540 cms. = 1 in.

58 cms. = X

$X = \frac{58}{2.540} \times 1$ inch

$= \frac{5800}{254}$

Ans. : 22.83 inches.

PROBLEMS

1. Convert 6 chains 20 yards 2 ft. to inches.
2. Convert 10 chains 21 yards 1 ft. to links.
3. Find the number of chains, feet and inches in 2,187 links.
4. How many inches are there in 43 links?
5. Convert 100 metres to yards.
6. How many centimetres are there in 23 inches?
7. Convert 235 centimetres to feet and inches.

2. SQUARE MEASURE CONVERSION

Ex. 1 :

How many square feet are there in 2,886 sq. inches?

1 sq. ft. = 144 sq. inches

X = 2,886 sq. inches

$X = \frac{2886}{144} \times 1$ sq. ft.

Ans. : 20.04 sq. ft.

Ex. 2 :

Convert 17,880 square feet to square chains and square yards.

Step 1 : To convert to square yards divide by 9 sq. feet (9 sq. ft. = 1 sq. yard)

$$9 \overline{)17880} = 1986.67$$

= 1986.67 sq. yds.

Put 0.675 sq. yards aside.

Step 2 : To convert to square chains divide by 484 sq. yards (484 sq. yds. = 1 sq. chain)

$$484 \overline{)1986} = 4.50/484$$

1936

50

Ans. : 4 sq. chains 50.67 sq. yards.

Ex. 3 :

Convert 615 sq. chains to acres.

1 acre = 10 sq. chains

∴ divide 10 into 615

Ans. : 61.5 acres.

Ex. 4 :

Convert $3\frac{1}{2}$ acres to square feet.

There are two ways to do this problem :

(a) By converting acres to square chains, square yards and then into square feet.

(b) By using the fact that there are 43,560 sq. feet in an acre.

If it is not possible to recall the number of sq. feet in an sacre it is best to do this problem using method (a).

Step 1 :
10 sq. chains = 1 acre
X sq. chains = $3\frac{1}{2}$ acres
X = 10 x 3.5 sq. chains
X = 35 sq. chains.

Step 2 :
484 sq. yards = 1 sq. chain
Y = 35 sq. chains
Y = 484 x 35 sq. yards
Y = 16,940 sq. yards.

Step 3 :
9 sq. ft. = 1 sq. yard
Z sq. ft. = 16,940 sq. yards
Z = 16,940 x 9 sq. ft.
Z = 152,460 sq. ft.

Ans. : 152,460 sq. ft.

CHECK by method (b).

Ex. 5 :

Given 1 hectare = 2.4710 acres. How many acres are there in 12 hectares?

1 hectare = 2.4710 acres
12 " = X
X = 12 x 2.4710 acres

Ans. : 29.652 acres.

Ex. 6 :

Convert 73 acres to hectares.

1 acre = 0. 404687 hectares

73 " = X

X = 73 x 0. 404687

Ans. : 29. 542 hectares.

This problem can also be worked out by saying :

1 hectare = 2. 4710 acres

X = 73 acres.

$$X = 73 \times \frac{1}{2.4710}$$

Ans. : 29. 542 hectares.

Ex. 7 :

Convert 2 square miles to square kilometres.

This problem can be worked out in many different ways depending on the conversion equivalents available. If the equivalent number of acres per hectare is known this problem can be solved by the fact that there are 640 acres in a square mile and 100 hectares in a square kilometre e.g.

Step 1 : 2 square miles = 2 x 640 acres

= 1280 acres

Step 2 : 1 hectare = 2. 4710 acres

X " = 1280 acres

$$X = 1280 \times \frac{1}{2.4710} \text{ hectares}$$

Step 3 : 100 hectares = 1 square kilometre

$$\frac{1200}{2.4710} \text{ hectares} = Y$$

$$Y = \frac{1280}{2.4710} \times \frac{1}{100} \text{ sq. kilometres}$$

Ans. : 5. 180 square kilometres.

Alternatively :

1 sq. mile = 2. 590 sq. kilometres

2 sq. mile = X

Ans. : 5. 180 square kilometres.

PROBLEMS

1. Convert 5812 sq. inches to square yards and square feet.
2. Convert 4896 sq. feet to sq. chains and sq. yards.
3. How many square feet are there in 4. 75 acres ?
4. Find the number of acres in 28. 5 hectares.
5. How many acres are there in 5. 5 square kilometres ?

3. CUBIC AND LIQUID MEASURE CONVERSIONS

Ex. 1 :

Find the number of cubic feet in 18, 785 cubic inches.

1 cubic foot = 1728 cubic inches

X cubic feet = 18, 785 cubic inches

$X = \frac{18785}{1728}$ cubic feet

Ans. : 10.871 cubic feet.

Ex. 2 :

Convert 218,650 cubic inches to cubic feet and cubic yards.

9 cubic ft. = 1 cubic yard

1,728 cubic inches = 1 cubic ft.

Step 1 : 1728 cubic inches = 1 cubic ft.

218,650 cubic inches = X

$X = \frac{218650}{1728}$ cubic ft.

X = 126.53 cubic ft.

Put 0.53 cubic ft. aside for the moment.

Step 2 : 27 cubic ft. = 1 cubic yard

126 cubic ft. = Y

$Y = \frac{126}{27}$ = 4 cubic yards 18 cubic ft.

Ans. : 4 cubic yards 18.53 cubic feet.

Ex. 3 :

How many gallons are there in 687 pints ?

8 pints = 1 gallon

687 pints = X

Ans. : 85.875 gallons.

Ex. 4 :

How many cubic metres in 70 cubic yards ?

1 cubic yard = 0.76455 cubic metres

70 cubic yards = X

X = 70 x 0.76455 cubic metres

Ans. : 54.5185 cubic metres.

PROBLEMS

1. Convert 17,832 cubic inches to cubic yards and cubic feet.
2. Convert 75 cubic feet to gallons.
3. How many cubic feet of space would 1000 gallons require ?
4. How many litres in 7.3 pints ?
5. How many cubic feet in 6250 gallons ?

4. WEIGHT CONVERSION

Ex. 1.

Convert 27,865 ounces to lbs.

1 lb. = 16 ounces

X = 27,865 ounces

$X = \frac{27865}{16}$ lbs.

Ans. : 1741.56 lbs.

Ex. 2 :

Convert 41,786 ounces to tons, hundredweight and pounds.

This problem must be done in a number of steps.

Step 1 : 1 lb. = 16 ounces

X = 41,786 ounces

$X = \frac{41786}{16}$ pounds

X = 2611.6 lbs.

Put the 0.6 lbs. aside for the moment.

Step 2 : 1 cwt. = 112 lbs.

X = 2,611 lbs.

$X = \frac{2611}{112}$ cwt.

X = 23 cwt. 35 lbs.

Put the 35 lbs. aside.

Step 3 : 1 ton = 20 cwt.

X = 23 cwt.

= 1 ton 3 cwt.

Ans. : 1 ton 3 cwt. 35.6 lbs.

Ex. 3 :

Convert 2 tons 5 cwt. to lbs.

1 ton = 2240 lbs.	∴	2 tons =	4480 lbs.
1 cwt. = 112 lbs.	∴	5 cwt. =	560 lbs.
		Total	5040 lbs.

Ans. : 5040 lbs.

Ex. 4 :

Convert 20 lbs. to kilograms.

1 lb. = 0.453592 kilograms

20 lbs. = X

X = 20 x 0.453592 kilograms

Ans. : 9.071840 kilograms

This could be done as follows :

1 kilogram = 2.2046 lbs.

X = 20 lbs.

$X = \frac{20}{2.2046}$

Ans. : 9.0719 kilograms

This is approximately the same as the above.

PROBLEMS

1. Convert 87,560 ounces to lbs. and then to cwt.
2. How many ounces are there in 2.3 tons?
3. How many grams are there in 5 tons?

5. MIXED MEASURE CONVERSION

Ex. 1 :

Convert 17 gallons of fresh water to its weight equivalent in pounds.

1 gallon = 10 lbs.

17 gallons = X

Ans. : 170 lbs.

Ex. 2 :

A tank contains 25 cubic feet of fresh water. What does it weigh in pounds?

1 cub. ft. = 62. 5 lbs.

25 cub. ft. = X

X = 25 x 62. 5 lbs.

Ans. : 1562. 5 lbs.

Ex. 3 :

What is the weight in kilograms of five gallons of water?

5 gallons = 5 x 10 lbs.

1 lb. = 0. 453592 kilograms

50 lbs. = X

Ans. : 22. 6796 kilograms.

Ex. 4 :

What is the weight of 2 cubic feet of water in kilograms?

Step 1 : 1 cubic ft. water = 62. 5 lbs.

2 cubic ft. water = X

X = 62. 5 x 2 lbs.

= 125 lbs.

Step 2 : 1 lb. = 0. 453592 kilograms

125 lbs. = Y

Y = 0. 453592 x 125 kilograms

Ans. : 56. 699 kilograms.

PROBLEMS

1. What would 3 cubic feet of fresh water weigh in pounds?
2. What would 7. 5 gallons fresh water weigh in kilograms?
3. A vessel contains 25 kilograms of water. What number of gallons does it contain?
4. A vessel contains 70 kilograms of water. How many cubic feet of space does the water occupy?

CHAPTER 10

PERIMETERS

The term "perimeter" is used to describe the line, real or imagined, enclosing an area.

This sheet of paper is enclosed by a perimeter. In this particular case the opposite sides of the area are of equal length and are straight lines. In the case of a farm paddock the perimeter may be irregular.

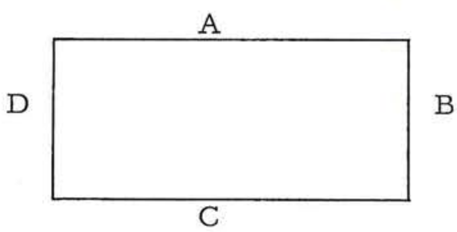
The method of obtaining the perimeter of various areas will be discussed under the following headings.

1. Rectangles
2. Squares
3. Triangles
4. Circles
5. Parallelograms
6. Trapeziums
7. Irregular or regular figures enclosed by straight or sinuous lines.

1. RECTANGLES

A rectangle is a 4 sided figure enclosing four right angles. The adjacent sides are generally unequal but opposite sides are equal.

In the diagram that side A is equal in length to side C and side B is equal in length to side D. The perimeter is equal to the sum of the lengths of sides A, B, C and D.



PERIMETER = A + B + C + D but
A = C and B = D
substituting A for C and B for D

PERIMETER = A + B + A + B
= 2A + 2B
or = 2 (A + B)

In practice A and B are replaced by L = LENGTH and B = BREADTH.

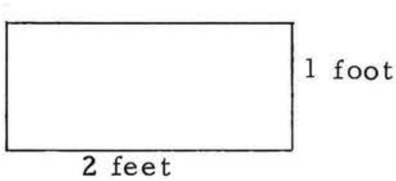
So the Perimeter = 2 (length + breadth)

or = 2 (L + B)

FORMULA : PERIMETER = 2 (L + B)

Ex. 1 :

What is the perimeter of a figure 2 feet long and 1 foot wide?



L = 2

B = 1

THE FORMULA :

PERIMETER = 2 (L + B)
Substituting for L and B = 2 (2 + 1)
= 2 x 3
= 6 feet

Ans. : Perimeter = 6 feet.

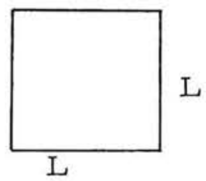
CHAPTER 10 continued :

PROBLEM

1. Find the perimeter of a rectangular paddock with sides of 8 and 6 chains.

2. SQUARES

A square is a special type of a rectangle in that it has sides of equal length.



The formula for the perimeter of a rectangle = 2(L + B)

In the case of a square L = B

Substituting for B in the formula

PERIMETER = 2 (L + L)

i.e. + 4 L

FORMULA : PERIMETER = 4L

Ex. 1 :

Find the perimeter of a square area with sides 15 chains long.

FORMULA : PERIMETER = 4L

Substituting = 4 x 15 chains

Ans. : Perimeter = 60 chains.

Ex. 2 :

A square has a perimeter of 40 feet. What is the length of each side?

FORMULA : PERIMETER = 4L

The perimeter is 40 feet.

$\therefore$ 40 = 4L

Cross multiply to get L on one side of the equation.

40 = 4L

$L = \frac{40}{4}$ feet

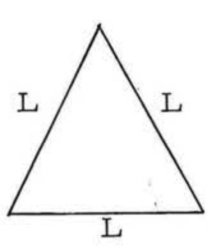
Ans. : L = 10 feet.

3. TRIANGLES

A triangle is a figure enclosed by 3 straight lines. The joining points of each two lines enclose an angle. All the angles may be equal and hence all the sides will be equal in length (equilateral triangle). Two angles may be equal and then two sides will be equal in length (isosceles triangle). One angle may be a right angle (90^{o}) in which case it is referred to as a right angle triangle.

Equilateral Triangle

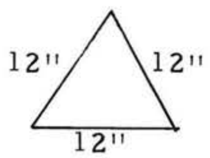
In the case of an equilateral triangle if the length of one side is known the perimeter can be found.



FORMULA : PERIMETER = 3L

Ex. 1 :

The length of a side of an equilateral triangle is 12''. What is its perimeter?



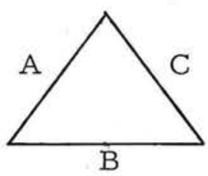
FORMULA : PERIMETER = 3L

= 3 x 12 inches

Ans. : Perimeter 36 ins.

Isosceles Triangle

In the case of isosceles triangle the length of all sides must be known.



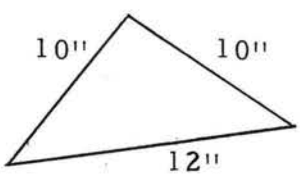
PERIMETER A + B + C

In this particular case A = C and the formula could be

PERIMETER = 2A + B

Ex. 2 :

The equal sides of a isosceles triangle are 10'' long. The other side is 12''. What is the perimeter?



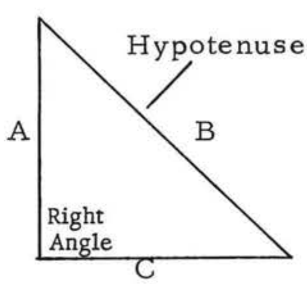
PERIMETER = 2A + B

Substituting in formula = 2 x 10 + 12 inches

Ans. : Perimeter = 32 inches.

Right Angled Triangle

In the case of a right angle triangle in order to determine the perimeter it is necessary only to know length of two sides. The third one can be calculated.



In the diagram side B is called the hypotenuse. A formula developed by PYTHAGORAS and bearing his name is used to determine the length of an unknown side when the length of the other two sides is known. In the diagram above the formula states that :

$B^2 = A^2 + C^2$

In words this means that the square of the length of the hypotenuse in a right angled triangle is equal to the sum of the squares of the length of the other two sides.

If B is unknown then $B = \sqrt{A^2 + C^2}$

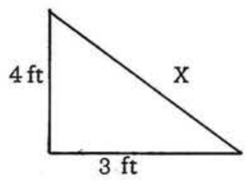
If C is unknown then $C = \sqrt{B^2 - A^2}$

and $A = \sqrt{B^2 - C^2}$

Ex. 3 :

The two sides enclosing the right angle in a right angle triangle are 4 ft. and 3 ft. respectively. Find the perimeter.

Step 1 : By Pythagoras $X = \sqrt{4^2 + 3^2}$



$= \sqrt{16 + 9}$

$= \sqrt{25}$

$X = 5$ feet

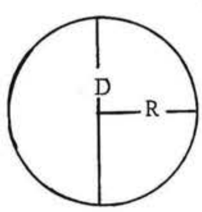
Step 2 : PERIMETER $= A + B + C$

$= 4 + 3 + 5$ ft.

Ans. : Perimeter = 12 feet.

There are other methods of finding the unknown length of a side of a triangle providing two sides and an angle, or one side and two angles are given. The formulas used need not concern students on this course. If required to find the unknown length of a side a suitable scaled diagram should be drawn with a protractor, ruler and compass. The length of the unknown side can then be measured off. Try this with Ex. 3 above.

4. CIRCLES



The perimeter of a circle is referred to as its circumference. The formula for circumference is :

FORMULA : CIRCUMFERENCE : π D or $2\pi R$

When π = APPROXIMATELY $\frac{22}{7}$

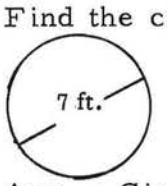
D = DIAMETER

R = RADIUS

$\therefore$ C $= \frac{22}{7} \times D$ or $\frac{44}{7} \times R$

Ex. 1 :

Find the circumference of a circle when the diameter is 7 feet.



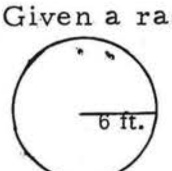
FORMULA : $C = \pi D$

Substituting $= \frac{22}{7} \times 7$ feet

Ans. : Circumference = 22 feet.

Ex. 2 :

Given a radius of 6 feet find the circumference of a circle.



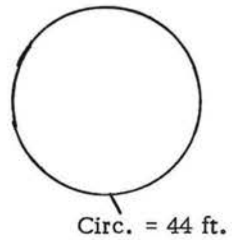
FORMULA : $C = 2\pi R$

Substituting values $= 2 \times \frac{22}{7} \times 6$

Ans. : Circumference = 37.71 ft. (approx.)

Ex. 3 :

The circumference of a tank is 44 feet. Find the diameter.



FORMULA : $C = \pi D$

Substituting known values

$$44 = \frac{22}{7} \times D$$

Cross multiply

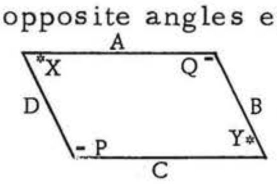
$$44 \times \frac{7}{22} = D$$

D = 14 feet

Ans. : Diameter = 14 feet.

5. PARALLELOGRAMS

A parallelogram is a quadrilateral with its opposite sides parallel and opposite angles equal.



In the diagram A = C and B = D

Angles X = Y and P = Q

To find the perimeter it is necessary to know the length of two adjacent sides. In this case A + B would have to be known.

Then :

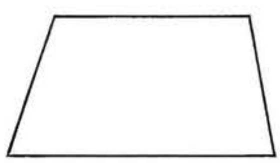
FORMULA : PERIMETER = 2A + 2B

If L and L' are substituted PERIMETER = 2 (L + L')

A square and a rectangle are special types of parallelograms.

6. TRAPEZIUMS

A trapezium is a quadrilateral with one pair of parallel sides.



To find the perimeter of a trapezium it is necessary to know the length of each side. To find the perimeter the best method is to measure all sides of the given figure.

7. IRREGULAR AND REGULAR FIGURES ENCLOSED BY STRAIGHT OR SINUOUS LINES

In the case of any difficult to calculate boundary one of two alternatives is available. Where an area has a number of straight lines delineating it a ruler can be used. For sinuous lines, such as are common around forest blocks on maps, a chartometer is recommended.

PROBLEMS

1. Find the length of the sides of a square block which has a perimeter of 48.8 chains.
2. The radius of a circular clearing is 10 chains, what is the circumference?
3. The diameter of a tree is 14". What is its girth? (A tree is often considered as having a circular cross section in which case the term girth can be substituted for circumference.)

4. A circular race track is 880 yards in length. What is its radius?
5. What is the perimeter of a right angled triangle with an hypotenuse of 13 feet and one side of 5 feet?
6. A right angled triangle has two sides of 3 feet and 4 feet. What is the length of the hypotenuse?

CHAPTER 11

AREAS

Area is defined as surface extent.

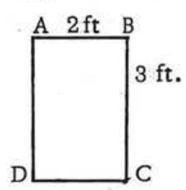
Irrespective of its shape, an area is bounded by a perimeter. A square is bounded by 4 equal length straight sides. A circular area is bounded by a continuous evenly curved line. Other areas do not have regular boundaries in which case the boundary is said to be irregular and the area enclosed is consequently irregularly shaped.

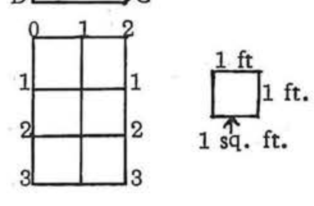
The subject of area measurement will be considered under the following headings :

1. Regular Shaped Areas
   - (a) Rectangles
   - (b) Squares
   - (c) Circles
   - (d) Parallelograms
   - (e) Trapeziums
   - (f) Triangles
2. Irregular Shaped Areas
   - (a) Polygons
   - (b) Areas bounded by irregular and sinuous lines.

1. (a) RECTANGLES

To determine the area of a rectangle it is necessary to know the length of two adjacent sides. The length can be given in units of one sort or another. For example an area could have sides of 3 feet and 2 feet. In the diagram side AB is 2 feet long and side BC 3 feet long.





If the perimeter is divided up into feet and the corresponding points on the opposite side joined a number of squares 1 foot long and 1 foot wide will be delineated.

Each unit 1 foot long and 1 foot wide is a square. Each square is said to be 1 square foot.

How many similar squares are there? The answer is 6. Therefore there are 6 square feet in an area 3 ft. long and 2 feet wide.

Notice this answer could also be reached by multiplying 3 ft. by 2 ft.

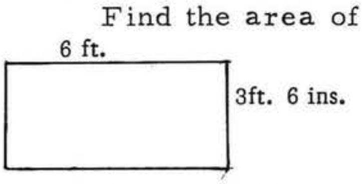
= 3 ft. x 2 ft. = 6 sq. ft.

In fact this is the method used to find the area of a rectangle.

FORMULA : AREA = L x B where L = Length, B = Breadth

Ex. 1 :

Find the area of a rectangular table 6 feet by 3 feet 6 ins.



3 ft. 6 ins. = 3. 5 ft.

Always calculate in one type of unit.

In this case feet should be used.

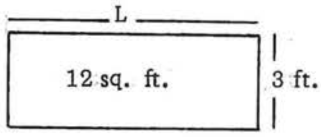
FORMULA : AREA = L x B

Substituting : Area = 6 x 3. 5 sq. ft.

Ans. : Area = 21 square feet.

Ex. 2 :

A rectangular area is 12 square feet in extent. Find the length when the breadth is 3 feet.



FORMULA : AREA = L x B

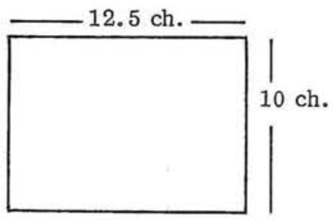
Substituting : 12 sq. ft. = L x 3 ft.

L = 12/3 ft.

Ans.: Length = 4 ft.

Ex. 3 :

A rectangular area is 10 chains long and 12.5 chains wide. What is its area in acres?



Step 1 :

FORMULA : AREA = L x B

Substituting = 10 x 12.5 sq. chains.

= 125 sq. chains

Step 2 :

10 sq. chains = 1 acre

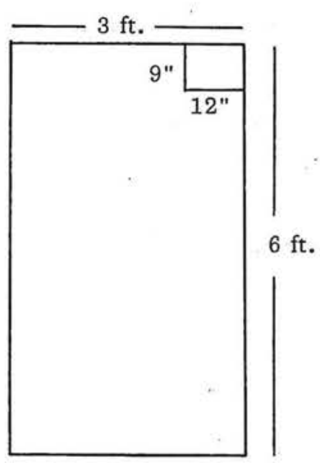
125 sq. chains = X

$X = \frac{125}{10} \times 1$ acre

Ans. : Area = 12.5 acres.

Ex. 4 :

A 6 ft. x 3 ft. sheet of 3 ply is cut up into backing boards 12" x 9". How many can be cut out of the sheet if no wastage is incurred?



Step 1 :

FORMULA = L x B

Area of sheet = 3 x 6 sq. ft.

= 18 sq. ft.

Step 2 :

Area of backing board

= 9/12 x 12/12 sq. ft.

= ¾ sq. foot.

Step 3 :

1 board = ¾ sq. ft.

X = 18 sq. ft.

$X = \frac{18}{3/4}$ boards

$= 18 \times \frac{4}{3}$ boards

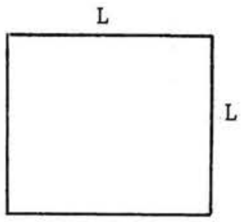
Ans. : No. of boards = 24

Is this answer really possible?

* Note : when dividing by a fraction invert the fraction and multiply by it.

1. (b) SQUARES

A square is a special case of a rectangle in that all its sides are of equal length. It is therefore only necessary to know the length of one side to determine its area.



$\text{AREA} = L \times B$ but $L = B$

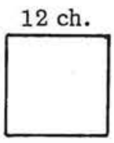
$\therefore \text{AREA} = L \times L$

$= L^2$

FORMULA : $\text{AREA} = L^2$

Ex. 1 :

A square paddock has one fence which measures 12 chains. How many acres are there in the paddock?



FORMULA : $\text{AREA} = L^2$

Substituting $= 12^2$ sq. chains

$= 144$ sq. ch.

Ans. : Area = 14.4 acres.

Ex. 2 :

A farm is square in shape and is 2250 acres in extent. What is the distance along one of the boundary fences?

FORMULA : $\text{AREA} = L^2$

Substituting : 2250 acres $= L^2$

$L^2 = 22500$ sq. chains

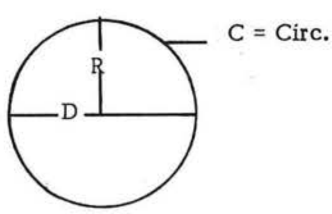
$L = \sqrt{22500}$

$L = 150$ chains.

Ans. : Length of one fence = 150 chains.

1. (c) CIRCLES

The formula for the area of a circle is :



(1) $\text{AREA} = \pi R^2$

Where π = approximately 22/7

and R = radius of the circle

Two other formulas can be used :

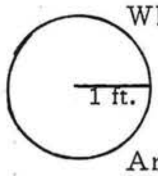
(2) When diameter is given

$\text{AREA} = \pi (D/2)^2$

or $\frac{D^2}{4}$

Ex. 1 :

What is the area of a disc of radius 1 ft.?



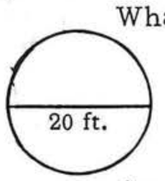
FORMULA : $\text{AREA} = \pi R^2$

Substituting $= 22/7 \times 1 \times 1$

Ans. : Area = 3.143 sq. ft.

Ex. 2 :

What is the area of an enclosure of diameter 20 feet?



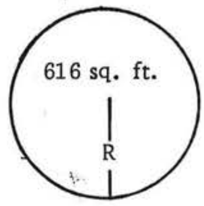
FORMULA : $\text{AREA} = \frac{\pi D^2}{4}$

Substituting $= 22/7 \times 20/4 \times 20/1$

Ans. : Area = 314.3 sq. feet.

Ex. 3 :

A circular area is 616 sq. feet in extent. What is its radius?



FORMULA : AREA = πR^2

Substituting : 616 = 22/7 R^2

R^2 = 616 x 7/22

R^2 = 196

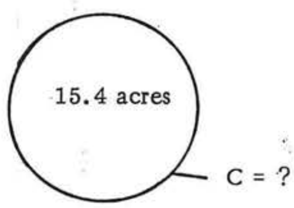
R = $\sqrt{196}$

R = 14 feet

Ans. : Radius = 14 feet.

Ex. 4 :

A circular area is 15.4 acres in extent. What is its circumference?



FORMULA : AREA = $C^2/4\pi$

Substituting and converting acres to square chains

154 = C^2 x 7/88

C^2 = 154 x 88/7

C = $\sqrt{22 \times 88}$

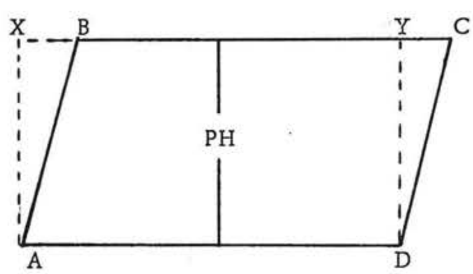
= $22^2 \times 2^2$

C = 44 chains

Ans. : Circumference = 44 chains.

1. (d) PARALLELOGRAMS

The relationship between a rectangle and a parallelogram can be seen in the diagram. ABCD is a parallelogram and AXYD a rectangle (right angled parallelogram).



Notice that the triangle AXB and DYC are exactly the same shape. The parallelogram looks as if it is the rectangle AXYD with a triangle cut off one end and put on the other. The triangle AXB is cut off one end of the rectangle and is joined on the other as triangle DYC. Then the area of the parallelogram is the same as that of the rectangle.

The formula for the area of a rectangle is

AREA = L x B

In this case call L the base and B the perpendicular height from the base.

Substituting into the formula :

AREA = BASE x PERPENDICULAR HEIGHT

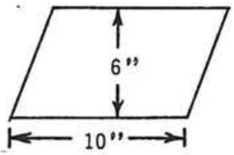
The area of the rectangle is equal to that of the parallelogram.

$\therefore$ FORMULA : AREA OF PARALLELOGRAM = B x PH

Where B = Base, PH = Perpendicular Height.

Ex. 1 :

A parallelogram has a base of 10'' and a perpendicular height of 6''. What is its area?



FORMULA : AREA = B x PH

Substituting values

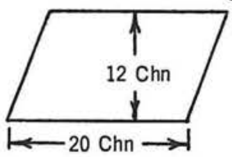
= 10 x 6 sq. inches.

Ans. : Area = 60 sq. inches.

Construct a parallelogram and a rectangle on the same base and find how they compare. Use the above units.

Ex. 2. :

An area is in the shape of a parallelogram. The shortest distance between the two longest sides is 12 chains and the longest side measures 20 chains. What is the acreage of the area?



FORMULA : AREA = B x PH

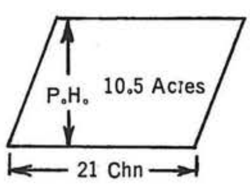
Substituting AREA = 20 x 12

= 240 sq. chains.

Ans. : Area = 24 acres.

Ex. 3 :

An area is in the shape of a parallelogram and has a long side of 21 chains. What is the shortest distance between the two long sides when the area is 10.5 acres?



FORMULA : AREA = B x PH

Substituting and changing acres to square chains

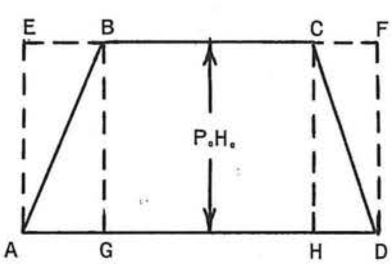
105 = 21 x PH

PH = 105/21 chains.

Ans. : Shortest distance between long sides = 5 chains.

1. (e) TRAPEZIUMS

A trapezium is a quadrilateral with only two sides parallel.



FORMULA : AREA OF A TRAPEZIUM

= PH x $\frac{1}{2}$AD + BC

or PH x $\frac{1}{2}$(A + B)

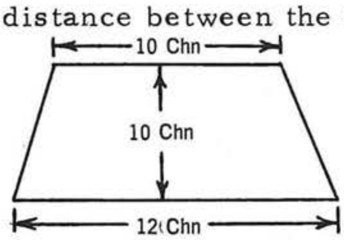
when A = Long Parallel Side

B = Short Parallel Side

PH = Perpendicular Height.

Ex. 1 :

A trapezium shaped field has parallel sides 10 and 12 chains long. The distance between the sides is 10 chains. What is the area in acres?



FORMULA : AREA = PH x $\frac{1}{2}$(A + B)

Substituting = 10 x $\frac{1}{2}$ x (12 + 10)

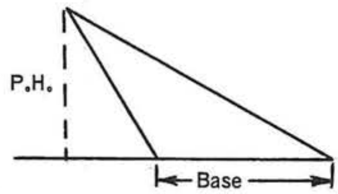
= 10 x 22/2

= 110 sq. chains.

Ans. : Area = 11 acres.

1. (f) TRIANGLES

In problem 1. (d) 3 the area of half a parallelogram was asked for. The problem is worked below.



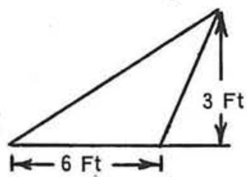
FORMULA :

AREA OF TRIANGLE = $\frac{1}{2}$Base x PH

when PH = Perpendicular Height.

Ex. 1 :

Find the area of a triangle with a base of 6 feet and perp. height of 3 ft.



FORMULA : AREA = $\frac{1}{2}$B x PH

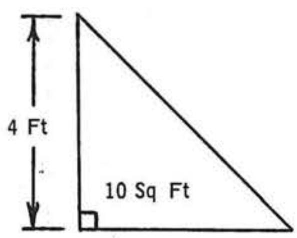
Substituting values = $\frac{1}{2} \times 6 \times 3$

= 9 sq. feet.

Ans. : Area = 9 sq. feet.

Ex. 2 :

A right angle triangle has an area of 10 sq. feet. One side is 4 feet long. What are the lengths of the other two sides?



Step 1 :

FORMULA : AREA = $\frac{1}{2}$B x PH

$10 = \frac{1}{2} \times 4 \times PH$

$PH = \frac{2 \times 10}{4}$ ft.

= 5 feet.

Step 2 :

Find hypotenuse by Pythagoras

X unknown $= \sqrt{5^2 + 4^2}$

$= \sqrt{41}$

= 6.4 feet approx.

Ans. : 5 feet and 6.4 feet approx.

2. (a) POLYGONS

A polygon can be either regular or irregular depending on whether all the sides are respectively equal or unequal. The methods of determining the area of a polygon are two.

(1) Divide a scale drawing of the polygon up into a series of triangles and then by measurement find the area.

(2) Use a planimeter or dot grid to take out the area. The planimeter is an accurate apparatus but at least 3 measurements should be made of each area. Any large discrepancy between readings should be checked. The dot grid should be used where slightly less accuracy is needed. At least 3 measurements should be taken of each area. The number of dots on the grid will influence the accuracy. The dot grid is generally a more speedy method of extracting areas than is the planimeter.

2. (b) AREAS BOUNDED BY IRREGULAR AND SINUOUS LINES

Either a dot grid or a planimeter should be used to assist in calculating the area.

PROBLEMS

1. A rectangular area is half a mile long and 1/8 mile wide. What is its area in acres?
2. A rectangular paddock is 33 chains long and has an area of 76 acres. What is its breadth in chains and yards?
3. How many five chains by four chains plots can be fitted into a regular paddock 16 chains by 35 chains? What is the area in acres of each plot and the area in acres of the paddock?
4. A paddock is in the shape of a square and encloses 160 acres. What is the length of a side?
5. What is the area of a roughly circular block of forest surrounded by a road 1.1 miles long?
6. What is the area of a circular disc with a diameter of 14 square inches?
7. What is the cross sectional area of a tree with a girth of 2'3"?
8. An area is in the shape of a parallelogram and is 6.25 acres in extent. What is the shortest distance between the long sides if the long side measures 12.5 chains?
9. Find the area of a triangular block of forest of base 45 chains and whose shortest distance from the base line to apex is 20 chains?

CHAPTER 12

VOLUMES

One of the dictionary definitions of "volume" is "solid content". Then volumetric measurement is the measurement of the solid, liquid or gaseous content of a structure. Volumetric content is expressed in cubic units e.g. cubic feet.

The following pages will deal with methods of determining volume under the headings.

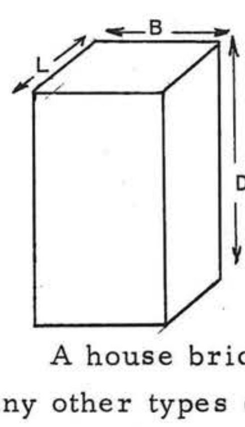
(1) Prisms
(2) Rectangular prisms
(3) Cubes
(4) Other types of prisms
(5) Pyramids
(6) Cylinders
(7) Cones
(8) Spheres
(9) Volumes of odd shaped structures

(1) PRISMS

A prism is a solid figure whose two ends are similar, equal and parallel rectilineal figures.

Sections on lineal, and area calculations have been covered. Each section has introduced a new dimension. Distance has a single dimension; area has two dimensions; and solids have a third dimension. The third dimension is called depth or thickness.

A house brick is a good example of a prism. Both ends have a similar shape. The two ends are connected by the other 4 sides of the brick which give the brick depth.



The formula for the surface of one end is = Length x Breadth. Then the Volume = Length x Breadth x Depth.

FORMULA : Vol. = L x B x D

where L = Length
B = Breadth
D = Depth

A house brick is an example of a rectangular prism. As there are many other types of prisms a formula to cover all types should be available. The type of prism depends on its end section. Therefore :

VOLUME = END AREA x DEPTH
= A x D

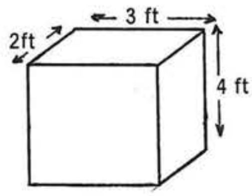
where V = volume
A = area
D = depth

(2) RECTANGULAR PRISMS

The following examples are concerned with the volume of rectangular prisms (already referred to above).

Ex. 1 :

Find the volume of a water tank 3 feet long 2 feet wide and 4 feet deep.



FORMULA V = A x D

= L x B x D

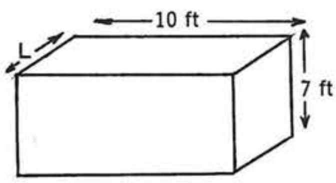
Substituting = 3 x 2 x 4 cu. feet

= 24 cubic feet.

Ans. : Volume = 24 cubic feet.

Ex. 2 :

A rectangular tank is 10 feet wide and 7 feet deep and contains 560 cubic feet of water when full. What is its length?



FORMULA V = L x B x D

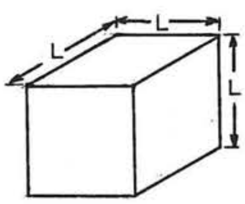
Substituting 560 = L x 10 x 7

$L = 560 \times \frac{1}{7} \times \frac{1}{10}$ feet

Ans. : Length = 8 feet

(3) CUBES

A cube is very similar to a rectangular prism but its length, breadth and depth are equal.



Formula for rectangular prism

V = A x D

= L x B x D

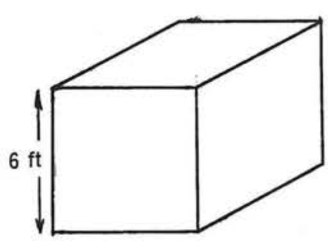
Substitute values for a cube

V = L x L x L

FORMULA : Volume of cube = L^3

Ex. 1 :

Find the volume in cubic yards of a cube shaped tank, which is 6 feet deep.



Step 1 :

FORMULA V = L^3

Substituting = 6 x 6 x 6 cubic feet.

Step 2 :

1 cubic yard = 27 cubic feet

X " " = 6 x 6 x 6 cu. ft.

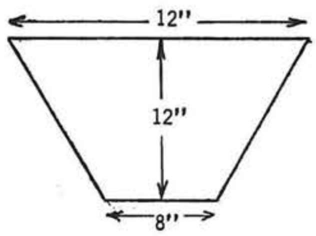
$X = \frac{6 \times 6 \times 6}{27}$ cubic yard

Ans. : 8 cubic yards.

(4) OTHER TYPES OF PRISMS

Ex. 1 :

A drain is 12" across at ground level and tapers to 8" wide at 1 foot below the surface. If the drain is 20 feet long what is the volume of earth removed from it?



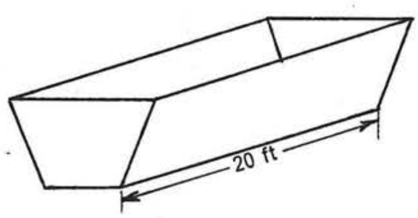
It is obvious that the drain is a trapezium in cross section.

Step 1 :

FORMULA : Area = $\frac{1}{2}$PH (A + B)

substituting values = $\frac{1}{2} \times \frac{12}{12} \times \frac{(12 + 8)}{12}$

area of cross section = $\frac{1}{2} \times \frac{1}{1} \times \frac{20}{12}$ sq. ft.



Step 2 :

FORMULA : Vol. = A x D

In this case D = 20 ft.

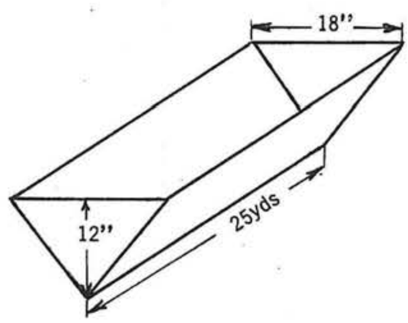
substituting values V = $\frac{1}{2} \times \frac{1}{1} \times \frac{20}{12} \times 20$ cu. ft.

= $\frac{50}{3}$ cu. ft.

Ans. : Volume = 16. 2/3 cu. ft.

Ex. 2 :

The cross section of a catch drain is triangular. The drain is 25 yds. long 18" wide at the top and 12" deep. What volume of earth (in cubic yards) was excavated during its construction?



Step 1 :

FORMULA : Area = $\frac{1}{2}$ B x PH

Substituting values = $\frac{1}{2} \times \frac{12}{36} \times \frac{18}{36}$ sq. yards

Area of cross section = $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2}$ sq. yards

Step 2 :

FORMULA : Vol. = A x D

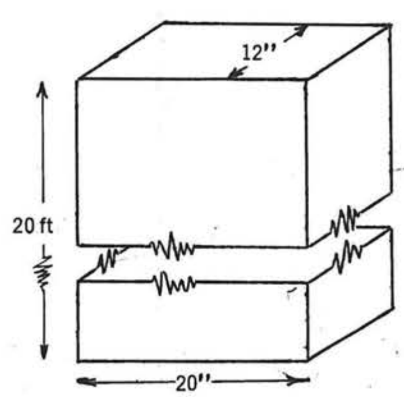
Substitute volume = $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} \times 25$ cu. yds.

= $\frac{25}{12}$ cu. yards.

Ans. : Volume = 2. 1/12 cubic yards.

Ex. 3 :

The cross section of a column is in the form of a parallelogram. The column is 20 feet tall and one side of the structure is 20". The distance through from this side to the opposite parallel side is 12". What volume of concrete went into the column?



Step 1 :

FORMULA : Area = B x PH

substituting = $\frac{20}{12} \times \frac{12}{12}$ sq. feet

Area of cross section $\frac{5}{3}$ sq. feet

Step 2 :

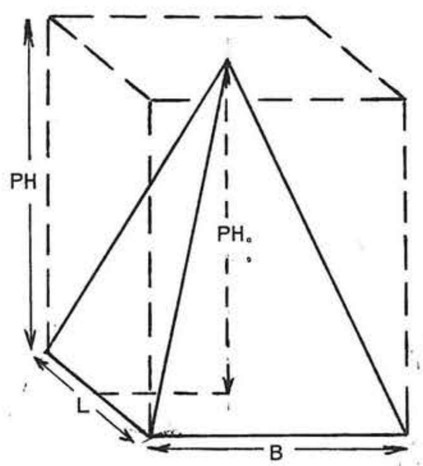
FORMULA : Vol. = A x D

substituting values = $\frac{5}{3} \times 20$ cu. ft.

Ans. : Volume = 33. 1/3 cubic feet.

(5) PYRAMIDS

A pyramid is a structure with usually a square base and sloping sides meeting at an apex. In all cases dealt with on this course the base will be rectangular unless otherwise specified.



The measurements required in order to find the volume of a pyramid are the length and breadth of the base and the perpendicular height from the base to the apex.

If a pyramid is constructed with its base area and height equal to those of a rectangular prism in which it is enclosed the volume of the pyramid is 1/3 of that of the rectangular prism. This gives the clue to the formula for volume of a pyramid.

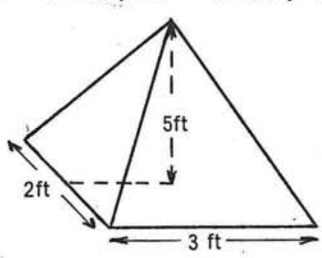
Volume of pyramid = 1/3 area base x perpendicular height.

FORMULA : V = 1/3 L x B x PH
or V = 1/3 A x PH

where A = area of base

Ex. 1 :

The dimensions of a pyramid are :
Base L = 3 ft., B = 2 ft., PH = 5 ft. What is its volume?

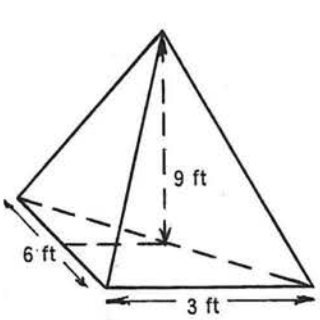


FORMULA : V = 1/3 L x B x PH
substituting = 1/3 x 3 x 2 x 5 cu. ft.

Ans. : Volume = 10 cubic feet.

Ex. 2 :

The base of a pyramid is in the form of a right angled triangle. The lengths of the sides enclosing the right angle are : 6 feet and 3 feet. The height of the pyramid is 9 feet. What is its volume in cubic yards?



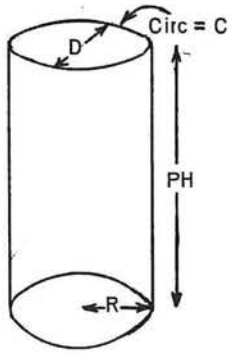
Step 1 :
Area Triangle A = $\frac{1}{2}$ B x PH
substituting = $\frac{1}{2}$ x 6 x 3 sq. ft.
= 9 sq. feet

Step 2 :
V = 1/3 A x PH
substituting = 1/3 x 9 x 9 cubic feet
= $\frac{27}{27}$ cubic yards

Ans. : Volume = 1 cubic yard.

(6) CYLINDERS

A cylinder is not unlike a prism in that it is a solid figure with its ends similar and parallel figures. The ends of a cylinder are circular. It then follows that the formula for the volume of a cylinder is :



VOLUME = A x D

Substituting the values in the diagram

FORMULA : $V = \pi R^2 \times PH$

or $= \frac{\pi D^2}{4} \times PH$

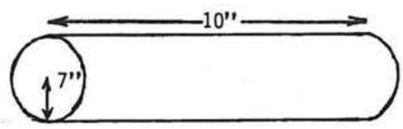
Where R = radius

D = diameter

PH = perpendicular height

Ex. 1 :

Find the volume of a cylinder of radius 7" and depth 10".



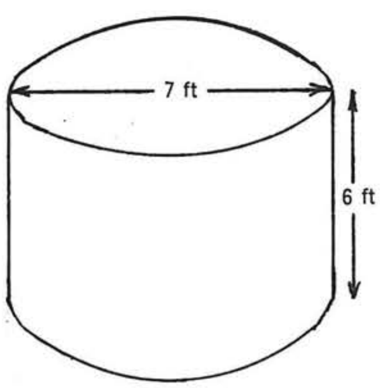
FORMULA : $V = \pi R^2 \times PH$

Substituting values $= \frac{22}{7} \times 7 \times 7 \times 10$

Ans. : 1,540 cubic inches.

Ex. 2 :

Find the volume in gallons of a cylindrical tank of diameter 7 feet and depth 6 feet.



Step 1 :

FORMULA : $V = \frac{\pi D^2}{4} \times PH$

Substituting values $= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{1} \times 6$ cu. ft.

= 231 cu. ft.

Step 2 :

1 cubic foot = $6\frac{1}{4}$ gallons

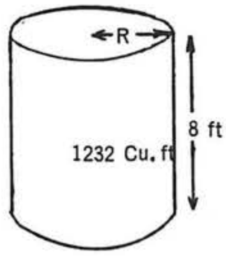
231 " " = X

$X = 231 \times \frac{25}{4}$ gallons

Ans. : 1,443¾ gallons.

Ex. 3 :

A tank contains 1,232 cubic feet of space and is 8 feet deep. What is its radius?



FORMULA : $V = \pi R^2 \times PH$

Substituting values $= 1232 = \frac{22}{7} \times R^2 \times 8$

cross multiplying $R^2 = \frac{1232}{1} \times \frac{7}{22} \times \frac{1}{8}$

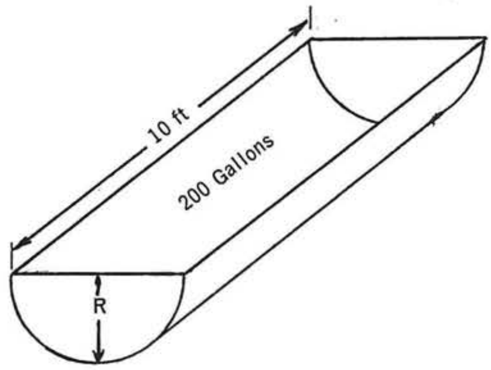
$R^2 = 7^2$

$R^2 = \sqrt{7^2}$

Ans. : Radius = 7 feet.

Ex. 4 :

A trough is in the form of half a cylinder and contains 200 gallons of water. It is 10 feet long. What is its radius?



Step 1 :

The volume of $\frac{1}{2}$ cylinder is 200 gals.

$\therefore$ volume of full cylinder is 400 gals.

1 cubic foot = $6\frac{1}{4}$ gallons

X = 400 "

X = $400 \times \frac{4}{25}$ cubic feet

Volume of cylinder = 64 cubic feet

Step 2 :

FORMULA : Volume = $\pi R^2 \times PH$

substituting values $64 = \frac{22}{7} \times R^2 \times 10$

$R^2 = 64 \times \frac{7}{22} \times \frac{1}{10}$

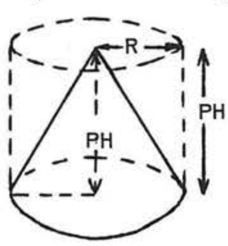
$= \frac{112}{55}$ feet

$= \frac{112 \times 12}{55}$ inches

Ans. : Radius = 24.4 inches.

(7) CONES

A cone has a circular base and tapers to an apex. The volume of a cone is 1/3 the volume of a cylinder on the same base and with the same height. (Compare with a pyramid). Then



FORMULA : Vol. = $1/3\ \pi R^2 \times PH$

or $= 1/3 \frac{\pi}{1} \times \frac{D^2}{4} \times \frac{PH}{1}$

where R = radius
D = diameter
PH = perpendicular height.

Ex. 1 :

Find the volume of a cone which has a base radius of 7" and perpendicular height of 6".

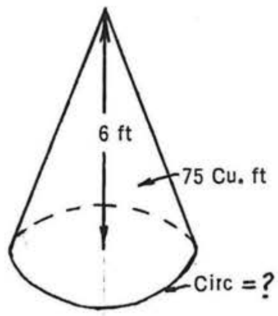
FORMULA : Vol. = $1/3 \pi R^2 \times PH$

substituting values $= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 6$ cu. in.

Ans. : 308 cubic inches.

Ex. 2 :

A cone is 6 feet tall and contains 75 cubic feet of space. What is the diameter of the base?



FORMULA : $V = 1/3 \frac{\pi D^2}{4} \times PH$

substituting values $75 = 1/3 \frac{D^2}{4} \times \frac{22}{7} \times 6$

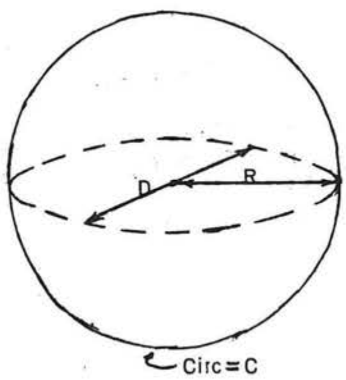
$D^2 = \frac{75}{1} \times \frac{7}{11}$

$C = 47\frac{8}{11}$

$= 7$ feet approx.

Ans. : Diameter of base 7 feet approx.

(8) SPHERES



The volume of a sphere is given by :

FORMULA : $Vol. = \frac{4\pi R^3}{3}$

where R = radius

Other formulas can be used also

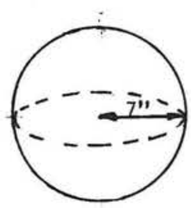
$V = \frac{1}{6}\pi D^3$

where D = diameter

This latter formula is derived by substituting $R = \frac{D}{2}$ in the formula $V = \frac{4\pi R^3}{3}$

Ex. 1 :

A sphere has a 7" radius. What is its volume?



FORMULA : $V = \frac{4\pi R^3}{3}$

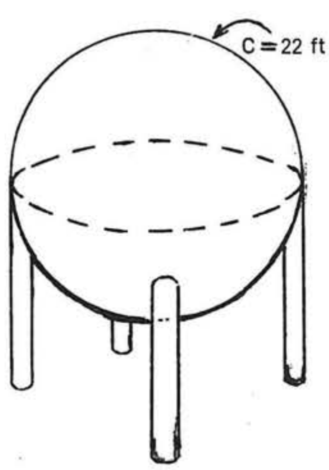
substituting : $V = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$ cu. ins.

$= \frac{4312}{3}$ cubic inches

Ans. : $1,437\frac{1}{3}$ cubic inches.

Ex. 2 :

A spherical liquid gas tank is filled. How many gallons does it hold if its diameter is 6 feet?



Step 1 :

FORMULA : $V = \frac{1}{6}\pi D^3$

substituting values $= \frac{1}{6} \times \frac{22}{7} \times \frac{6}{1} \times \frac{6}{1} \times \frac{6}{1}$

$= \frac{792}{7}$ cubic feet

Step 2 :

1 cubic foot $= 6\frac{1}{4}$ gallons

$\frac{792}{7}$ cu. ft. = X

$X = \frac{792}{7} \times \frac{25}{4}$

Ans. : Cubic content = 707.1 gallons.

(9) VOLUMES OF ODD SHAPED STRUCTURES

The cubic content of irregular shaped bodies cannot be determined by use of the previous formulas. In some cases mathematical methods are available but they are outside the scope of this course.

In forestry the major volumetric measurement is concerned with trees and logs. Most trees and logs do not fit into the shapes so far described. However, they do approach the shape of a cylinder or a cone. Further reference will be made to tree and log volume determination by measurement later in this course and in the mensuration course. (Ref.

One method which can be used to give accurate volume measurement of irregular solids is to measure the volume of fluid displaced by the object when completely immersed. The volume of displaced fluid is equal to the volume of the immersed object.

PROBLEMS

1. Find the volume of a drain 25 yards long 3 feet wide and 18" deep. Express your answer in cubic feet.
2. In designing a water tank to hold 600 gallons how deep must it be if it measures 6 feet by 10 feet? (6.25 gallons = 1 cu. ft.)
3. A water tank is shaped like a cube and contains 512 cubic feet. What are its dimensions?
4. Find the capacity in gallons of a trough 10 ft. long, 2 ft. wide at the top, 1 ft. wide at the base and 15" deep.
5. A "V" drain is cut beside a road and the material carted away. The drain is 60 feet long, 18" wide and 15" deep. How many cubic yards of soil are shifted?
6. Find the volume of a pyramid which has a base of 6 ft. by 4 ft. and which is 7 ft. high.
7. How many gallons can be stored in a tank which has a diameter of 6 feet and a depth of 5 ft. 6 ins. ?
8. A cylindrical log contains 4 load of timber. If the log is 20 ft. long, what is its diameter?
9. What is the volume in cubic yards of a conical tank 12 ft. high and 14 ft. in diameter? How many gallons would this tank hold?
10. Find the volume of a spherical tank of radius 10 feet.

CHAPTER 13

SURFACE AREAS

Methods used to determine the surface area of solid figures are covered under the following headings.

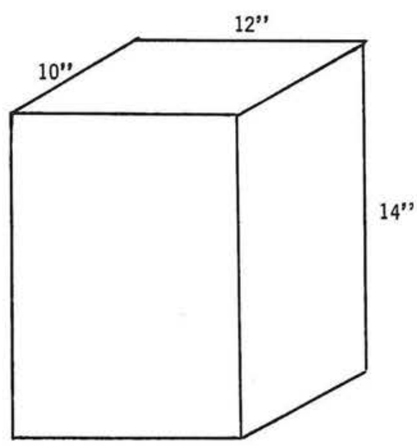
(1) Rectangular prisms
(2) Cubes
(3) Pyramids
(4) Cylinders
(5) Cones
(6) Spheres

(1) RECTANGULAR PRISMS

A rectangular prism has 3 sets of surfaces. One set constitutes the top and bottom and 2 sets, the other four sides. The two surfaces making up each set are identical in shape and area. It is therefore only necessary to calculate the area of one surface in each set and double it to find the set area.

Ex. 1 :

Calculate the surface area of a rectangular prism, 12" long, 10" wide and 14" deep.



FORMULA : A = L x B

Set 1 :

Area of set is twice the area of one end

2A = 2(L x B)

= 2(10 x 12)

= 240 sq. inches.

Set 2 :

= 2(10 x 14)

= 280 sq. inches.

Set 3 :

= 2(14 x 12)

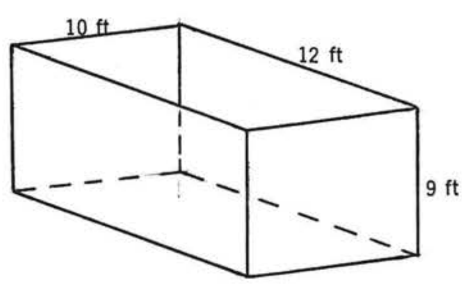
= 336 sq. inches.

Surface area = Set 1 + Set 2 + Set 3 = 240 + 280 + 336 sq. inches.

Ans. : Surface Area = 856 sq. inches.

Ex. 2 :

A room is to be painted out. It is 12 feet long, 10 feet wide and the ceiling 9 feet from the floor. What area in square feet must be covered?



In this particular case the surface area of only 5 instead of 6 sides is required.

FORMULA : A = L x B

Set 1 :

Area of one set of walls

= 2(9 x 10) sq. ft.

= 180 sq. feet.

Set 2 :

Area of second set of walls

= 2(12 x 9) sq. ft.

= 216 sq. feet

Ceiling

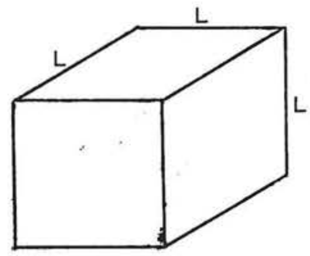
= 12 x 10 sq. ft.

= 120 sq. feet

Ans. : Total area to be painted = 516 sq. feet

(2) CUBES

A cube is a prism with all sides equal in area.



Then

Surface Area = 6 times the length by the breadth of one surface

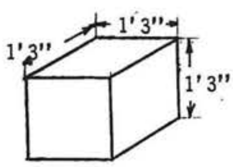
Surface Area = 6 x L x B

$= 6\ L^2$

FORMULA : Surf. Area = $6L^2$

Ex. 1 :

Find the surface area of a cube which has one side measuring 1'3".



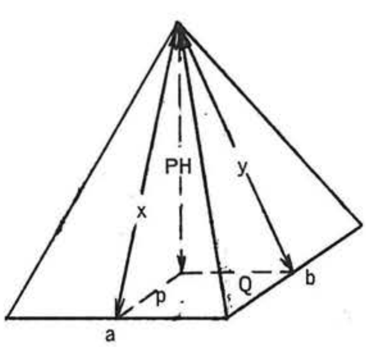
FORMULA : Surface Area = $6L^2$

substituting = $6 \times \frac{5}{4} \times \frac{5}{4}$ sq. ft.

Ans. : Surface Area = $9\frac{3}{8}$ sq. feet.

(3) PYRAMIDS

In the case of pyramids the surface area of the base is usually relatively simple to calculate. However, the sloping sides must be considered as triangles and their areas calculated as such.



Step 1 :

The base area is calculated using the formula A = L x B

Step 2 :

The surface area of sets of opposite sides must be calculated as triangles. However, the perpendicular height of the triangle or the slant height (S.H.) is not known. The perpendicular height of the pyramid and the lengths of the sides a and b are known.

By Pythagoras the slant height can be calculated.

In the diagram then $X = \sqrt{P^2 + PH^2}$ but $P = \frac{b}{2}$

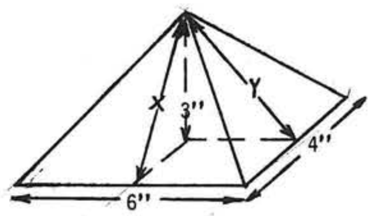
so $X = \sqrt{\frac{b^2}{4} + PH^2}$

similarly $Y = \sqrt{\frac{a^2}{4} + PH^2}$

Once X and Y have been calculated the surface areas can be calculated. An example will clarify the procedure.

Ex. 1 :

A pyramid shaped solid has a base 6'' x 4'' and a perpendicular height of 3''. What is its surface area?



Step 1 :

Base FORMULA : = L x B
= 6 x 4 = 24 sq. inches.

Step 2 :

Find slant height.

$X = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

= 3.60'' approx.

$Y = \sqrt{3^2 + 3^2}$

$= \sqrt{18}$

= 4.24'' approx.

Step 3 :

Set 1 — Surface Area = 2 ($\frac{1}{2}$B x SH)

where S.H. = slant height

substituting = 2 x $\frac{1}{2}$ x 6 x 3.60

= 21.60 sq. inches

Set 2 — = 2 x $\frac{1}{2}$ x 4 x 4.24

= 16.96 square inches

Step 4 :

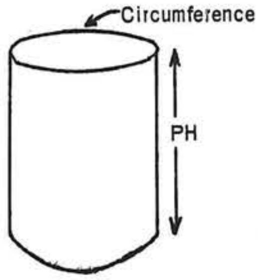
Total up areas

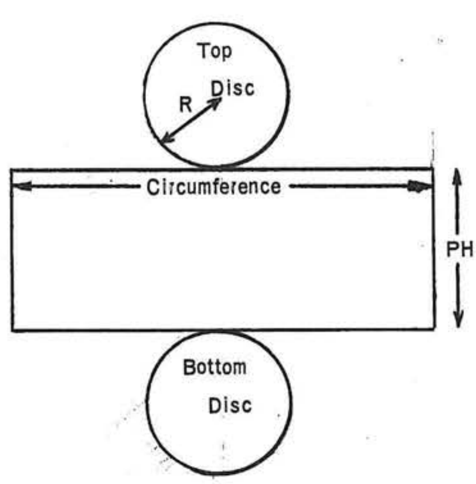
= 24.00 + 21.60 + 16.96 sq. inches.

Ans. : Surface Area = 62.56 sq. inches.

(4) CYLINDERS

When calculating surface areas it is best to consider a cylinder as two circular discs connected by a hollow tube. If the tube is cut down one side and flattened out the result is as in the diagram.





Then the calculation must be done in two parts.

Step 1 : For the discs.

FORMULA : Area = πR^2

There are two discs therefore the area of both = $2\pi R^2$

Step 2 :

The curved surface when flattened has as one of its measurements the circumference of the cylinder and as the other the perpendicular height of the cylinder.

Then

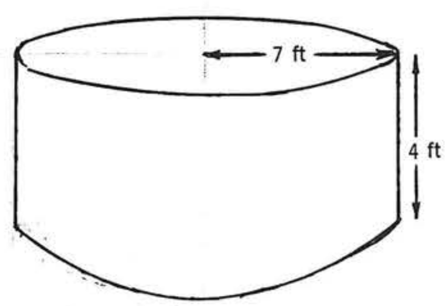
FORMULA : Curve Surface Area = $2\pi R \times PH$

To get the total areas add the two together.

Total surface area = $2\pi R^2 + 2\pi R \times PH$

Ex. 1 :

What is the total surface area of a tank 4 feet high and 7 feet in radius ?



Step 1 :

Area of ends

FORMULA : Area = πR^2

2 ends = $2\pi R^2$

substituting = $2 \times \frac{22}{7} \times 7 \times 7$ sq. ft.

= 308 sq. feet

Step 2 :

Curved surface

FORMULA : Area = $2\pi R \times PH$

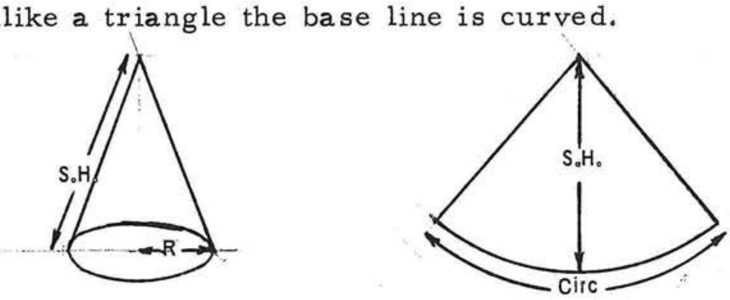
substituting = $2 \times \frac{22}{7} \times 7 \times 4$

= 176 sq. feet

Ans. : Surface Area 484 sq. feet.

(5) CONES

When a cone is cut along the shortest straight line from base to apex and is flattened out the resulting figure is very similar to a triangle. However, unlike a triangle the base line is curved.



The formula which gives a close approximation of the curved surface looks somewhat similar to the formula for the area of a triangle.

Curved Surface Area = $\frac{1}{2}$ circumference x slant height

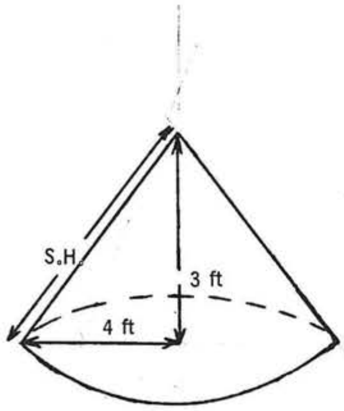
FORMULA : Curved Surf. Area = $\pi R \times S.H.$

The stand height must be calculated from the radius and the perpendicular height of the cone.

If the base of the cone is sealed the surface area of the base also must be calculated.

Ex. 1 :

Find the area of the curved surface of a cone which has a radius of 4 feet and perpendicular height of 3 feet.



Step 1 :

Find the slant height

$S.H. = \sqrt{4^2 + 3^2}$

$= \sqrt{25}$

$= 5$ ft.

Step 2 :

FORMULA : Curved Surface Area = $\pi R \times SH$

Substituting values $= \frac{22}{7} \times 4 \times 5$ sq. ft.

$= \frac{440}{7}$ sq. ft.

Ans. : 62.857 sq. feet.

(6) SPHERES

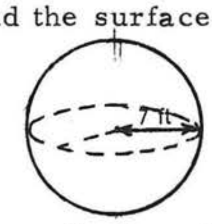
The formula for the surface area of a sphere looks very similar to that for a circular area. The relationship will make the formula easier to remember.

FORMULA : Surface Area = $4\pi R^2$

The surface area of a sphere is 4 times that of a circular area of the same radius.

Ex. 1 :

Find the surface area of a spherical tank of radius 7 feet.



FORMULA : Surface Area = $4\pi R^2$

Substituting values $= 4 \times \frac{22}{7} \times 7 \times 7$ sq. ft.

Ans. : Surface Area = 616 sq. feet.

PROBLEMS

1. Find the surface area in square feet of a rectangular prism 20'6" by 6 yards wide and 11'3" deep.
2. Find the surface area in square yards of a cube with a side 3'6" long.
3. Find the surface area of a pyramid with base 6 ft. by 6 ft. and 4 ft. high.
4. Find the surface area in square yards of a pyramid 40 feet high and a 60 feet by 50 feet regular base.
5. What is the surface area in square yards of a cylinder 7 ft. tall and 5 ft. in diameter?
6. The total surface area of a tank is 880 sq. ft. If its radius is 10 feet, what is its perpendicular height?

7. Find the curved surface of a cone 8 feet high and with a diameter of 8 feet.
8. The surface area of a cone is 75 sq. ft. and the radius is 5 feet. What is the perpendicular height?
9. Find the surface area in square feet of a sphere of radius 14".
10. A sphere has a surface area of 126 sq. ft. What is its radius?

CHAPTER 14

PERCENTAGE, DISCOUNT, PROFIT AND LOSS

In Chapter 7 as an example of proportion the quantities of blue metal, sand and cement in a batch of concrete was used. If the ratio is 4 : 2 : 1, what fraction of the mix is cement? The answer is 1/7.

This means that 1 part in 7 is cement. If a batch of concrete is made up of a 100 units metal, sand and cement, how many units would be cement.

1 unit cement in 7 units concrete

X units " " 100 " "

$X = \frac{100}{7}$ units cement

= 14.3 units cement approx.

Then there are 14.3 units per 100. The Latin word for 100 is "centum".

Then there are 14.3 units per centum. From this point it is easy to see how the terms percent and percentage were derived. In the above example the percentage of cement in the mix is 14.3 percent. Abbreviated to an easily expressed mathematical form this is written as 14.3%.

The method used to find percentage is to multiply the fraction by 100.

FORMULA : Percent = Fraction x 100

Ex. 1 :

It is found that 25 out of 100 eggs are bad. What percentage does this represent?

FORMULA : Percent = Fraction x 100

$= \frac{25}{100} \times 100\%$

Ans. : 25%

Ex. 2 :

Of a batch of 2,650 batteries 4% are faulty. How many are faulty?

FORMULA : Percent = Fraction x 100

substituting values $4 = \frac{X}{2650} \times 100$

$X = 4 \times \frac{2650}{100}$

Ans. : 106 faulty batteries.

Ex. 3 :

A log is sawn into timber. In the process off-cuts and sawdust are produced. The ratio of products is timber to off-cuts to sawdust = 4 : 3 : 3. What percentage went to each end product?

FORMULA : Percent = Fraction x 100

Timber

substituting percent $= \frac{4}{10} \times 100\%$

= 40%

Off-cuts

percent $= \frac{3}{10} \times 100\%$

= 30%

Sawdust percent = as for off-cuts

Ans. : 40% timber, 30% off-cuts, 30% sawdust.

Ex. 4 :

Five students do an exam. Find the average class mark in percentage when the marks are $6\frac{1}{2}$, 5, 4, 8 and $7\frac{1}{2}$ in each case out of a possible of 10.

	$6\frac{1}{2}$
	5
	4
	8
	$7\frac{1}{2}$
Total	31
Average	6.2

FORMULA : Percent = Fraction x 100

substituting = $\frac{6.2}{10} \times 100\%$

Ans. : Average = 62%

The use of percentage is important in problems involving discounts, profit and loss. Consider the following problems.

Discounts

In problems involving discounts, we are usually given two of the following (i) marked price (ii) discount price and (iii) % discount, and we are required to find the unknown.

Example 5 :

The marked price of an article is $10 but is offered for sale at a discount of 10%. What is the discount price?

To solve this problem we can assume that the marked price is equivalent to 100%. Then since the discount is 10%, the discount price will be 90% of the marked price.

This information can be set out as follows :

marked price = $10 = 100%

discount price = $X = 90%

The unknown discount price ($X) can be found by cross multiplication.

i.e. $10 x 90% and $X x 100%

which is $10 \times 90 = X \times 100$

divide both sides by 100 then $X = \frac{10 \times 90}{100}$

$X = \$9$

Ans. : $9 which is the discount price.

Example 6 :

The marked price of an electric shaver is $24, but it is offered for sale at $21. What % discount is received?

The information can be set out as follows :

marked price = $24 = 100%

discount price = $21 = X%

by cross multiplication $24 \times X = 21 \times 100$

dividing both sides by 24 $X = \frac{21 \times 100}{24}$

$X = 87\frac{1}{2}\%$

So \$21 is $87\frac{1}{2}\%$ of the marked price of \$24 and the discount % is therefore $12\frac{1}{2}\%$.

Profit and Loss

In problems involving the calculation of profit and loss exactly the same calculation is used but instead of being given the marked price and the discount price, cost price and selling price is used and cost price is equated to 100%.

Example 7 :

The cost price of a car tyre is \$20 and a salesman sells it for \$23, what % profit does he make?

The information can be set out as follows :

cost price = \$20 = 100%

selling price = \$23 = X%

By cross multiplication $20 \times X = 23 \times 100$

Dividing both sides by 20 $X = \frac{23 \times 100}{20}$

$= 115\%$

So the selling price of \$23 is 115% of the cost price of \$20.

The profit % is therefore 15%.

Example 8 :

A refrigerator which costs a firm \$180 is sold at a loss of 15%. What is the selling price? The information can be set out as follows :

Cost Price = \$180 = 100%

Selling Price = \$X = 85%

By cross multiplication $100 = 180 \times 85$

$X = \frac{180 \times 85}{100}$

$X = \$153$

Ans. : Selling Price = \$153.

These examples are simple but consider the following problem.

Example 9 :

A wood lathe is marked for sale at \$88 and if sold for this price would have brought a profit of 10%. However it is sold for \$68 - what % profit or loss did the firm make on the lathe?

Since \$88 represented a profit of 10% on the cost price, it is 110% of the cost price.

The information can then be set out as follows :

marked price = \$88 = 110%

selling price = \$68 = X%

by cross multiplication $88 \times X = 68 \times 110$

$X = \frac{17 \times 10}{2}$

$= 85\%$

i.e. the selling price is 85% of the cost price and the firm makes a 15% loss.

The method used in this problem is just the same as the examples above and should present no problem as long as you remember that the cost price is always equal to 100%.

PROBLEMS

1. Out of a sample of 160 pine seedlings, 25 are substandard and are discarded. What percentage does this represent?
2. In an area of 7, 500 acres $12\frac{1}{2}\%$ is forested, 20% is part cleared and $17\frac{1}{2}\%$ is swamps and stone. The remainder is cleared. What acreage falls into each classification?
3. A sawmill sells \$158. 20 worth of timber but gives a 5% discount for cash. How much would the mill receive for a cash sale?
4. An electric fry pan marked for sale at \$35. 50 is offered for sale at \$24. 85 (i) What percent discount is received (ii) What would the buyer pay if the discount was 10%?
5. If it costs a mill \$172. 80 to produce 5 loads of timber, for how much must it sell this timber to make a profit of $12\frac{1}{2}\%$?
6. A car is sold for \$414 at a loss of 8%. What profit would the dealer have made if the car was sold at a profit of 6%?
7. The cost price of a T.V. set is \$150 and a firm wishes to make 22% profit. However a discount of 5% is offered for cash. In giving this discount how much less does the firm make than if it made the full profit of 22%?

CHAPTER 15

SIMPLE AND COMPOUND INTEREST

In dealing with problems on simple and compound interest the terms which are used must first be explained.

The interest rate is the % interest charged for a given period of time, usually a year. For example an interest rate of 6% per annum. Although interest rates are usually charged on a per annum basis this need not necessarily be so and the interest rate could be say 2% per month.

The principal is the capital amount on which interest is charged. If a man invests \$500 in a building society at an interest rate of 6% per annum the principal is the \$500, the capital which has been invested.

Simple Interest

When interest is calculated on the same principal over the whole period of the loan it is called simple interest.

Example 1 :

Find the S.I. on \$200 invested for 2 years at 5% per annum.

To solve this problem find the interest payable for one and multiply this amount by the number of years.

$$\text{S.I.} = \$200 \times \underbrace{\frac{5}{100}}_{\text{interest rate}} \times 2 \text{ years} = \$20$$

Even if the loan is being paid off over the period, if simple interest is charged, it is calculated on the same principal over the whole period.

Example 2 :

A faller borrows \$200 to buy a chain saw and undertakes to pay it back at 10% simple interest over four years. How much interest will he pay and how much must he pay off each year.

$$\text{S.I.} = \$200 \times \frac{10}{100} \times 4 \text{ yrs.}$$

$$= 80$$

$\therefore$ Total debt to be paid off is Principal + S. Interest = 200 + 80
= \$280

Amount to be paid off each year over 4 years is then

\$280 ÷ 4 yrs. = \$70

If two of the following are given, we can find the unknown. Principal, interest rate, amount of interest.

Example 3 :

At 5% per annum simple interest a sum of money earnt \$780 in interest in 4 years.

Find the principal.

Let the principal = \$X

then we know that $X \times \frac{5}{100} \times 4 \text{ yrs.} = \$780$

Transferring $\frac{5}{100}$ and 4 to the other side of the = sign we must invert them and we then have

$$X = 780 \times \frac{100}{5} \times \frac{1}{4}$$

$$X = \$3,900$$

Ans. : Principal = \$3,900

Example 4 :

A sum of money will increase in value from $650 to $806 in 4 years if invested in a company - what would be the simple interest rate?

The amount of interest earnt is 806 - 650 = $156

Let the interest rate = X%

Then we know $650 \times \frac{X}{100} \times 4 = 156$

Transfering $650, $\frac{1}{100}$ and 4 to the other side of the = sign we must invert them giving $X = 156 \times \frac{1}{650} \times 100 \times \frac{1}{4}$

$X = 6\%$

Ans. : Interest Rate = 6%.

Compound Interest

In problems involving compound interest the interest is added to the principal at the end of each time period which is usually a year, but may be a month or a day.

Example 5 :

Find the C.I. on $5,000 for 2 years at 4% per annum.

Thus we must calculate the interest obtained in each year and add this to the principal to obtain the principal for the calculation of the next year's interest.

The interest for year 1 will be :

$5000 \times \frac{4}{100} \times 1$ yr. = $200

Thus at the end of year 1 the principal will be $6000 + $200 = $6,200.

The interest for year 2 will be $\frac{6200}{1} \times \frac{4}{100} \times 1$ yr. = $248

Thus the amount of Compound Interest earnt in two years on $5,000 is $200 + $248 = $448.

Example 6 :

What will be the value of an investment of $600 after 3 years at 5% per annum compound interest?

Interest for year 1 is $600 \times \frac{5}{100} \times 1$ yr. = $30

Value of investment after year 1 = $600 + $30 = $630

Interest for year 2 is $630 \times \frac{5}{100} = \frac{315}{10}$ = $31.50

Value of investment after year 2 = $630 + $31.50 = $661.50

Interest for year 3 is $661.50 \times \frac{5}{100}$ = $33.075

Value of investment after year 3 = $661.50 + $33.07 = $694.57

Ans. : $694.57

You will note that because the principal increases each year the amount of interest earnt also increases and there is a "snowballing" effect of the amount of interest earnt.

When a debt is being reduced on compound interest the principal becomes smaller each year, by the amount paid off. The total amount of interest paid is therefore smaller than would be the case if simple interest was paid.

Example 7 :

A faller borrows $200 to buy a chainsaw and undertakes to pay the principal back in equal instalments over four years. If compound interest at 10% is due how much interest will he pay and how much will he pay each year?

Interest for year 1 is $200 \times \frac{10}{100} \times 1 = \$20$

Amount repayable in year 1 is one fourth of the principal + interest

$= \$50 + \$20 = \$70$

Interest for year 2 - since the principal is now $150 is

$150 \times \frac{10}{100} = \$15$

Amount repayable in year 2 is $50 + $15 = $65

Interest for year 3 - principal is $100

$100 \times \frac{10}{100} = \$10$

Amount repayable in year 3 is $50 + $10 = $60

Interest for year 4 = principal is

$50 \times \frac{10}{100} = \$5$

Amount repayable in year 4 is $50 + $5 = $55

Thus the total interest payable is $20 + $15 + $10 + $5 = $50

Compare this $50 paid at compound interest rates with $80 (4 yrs. x $20) which would be paid at simple interest rates.

PROBLEMS

1. Find the simple interest on $850 invested for 2 years at $3\frac{1}{2}$% per annum.
2. What will $625 amount to in $5\frac{1}{2}$ years at 4% per annum simple interest?
3. At 5% per annum the simple interest on a sum of money in 3 years was $97.50. Find the principal?
4. At 6% per annum simple interest a sum of $880 yielded $184.80. For how long was it invested?
5. Find the compound interest on $6,000 for two years at 4% per annum.
6. What will $2,570 amount to in 2 years at $2\frac{1}{2}$% compound interest?
7. What would be the difference between simple and compound interest payable on a loan of $1,500 for 2 years at 4% per annum?

CHAPTER 16

UNITS OF TIMBER MEASURE AND CALCULATION OF LOG VOLUME

Although the calculation of timber volume is a simple arithmetical problem which anyone can solve given time, quick and accurate calculation using the correct method saves time.

UNITS OF TIMBER MEASURE

1. Cubic foot - a block of wood 12" x 12" x 12".
2. Load - 50 cubic feet of timber. This unit is only used in Western Australia.
3. Superficial foot - usually abbreviated to super foot is a piece of timber 12" x 12" x 1".
4. Lineal or Running feet - The number of feet of timber of a certain cross section, e.g. 12 feet of 6 x 2" is 12 lineal feet of 6 x 2" timber.

CONVERSION OF UNITS

Example 1 :

How many cubic feet are there in 75 super feet?

1 cubic foot = 12 super feet

X " " = 75 super feet

$$X = \frac{75}{12}$$

Ans. : $6\frac{1}{4}$ cubic feet.

Example 2 :

How many super feet are there in 6.25 loads?

1 load = 50 x 12 super feet

6.25 loads = X super feet

X = 6.25 x 50 x 12

Ans. : 3,750 super feet.

CALCULATION OF THE TIMBER VOLUME OF LARGE ORDERS

When there are many pieces of timber of the same cross section convert to lineal feet before calculating the volume. This saves time rather than summing the volume of each individual piece of timber.

Example 3 :

How many cubic feet of timber in the following order?

Cross Section	Length	No. of Pieces	Linear Ft.
6 x 2	18 ft.	10	180
6 x 2	16 ft.	8	128
6 x 2	10 ft.	10	100
4 x 3	18 ft.	12	216
4 x 3	12 ft.	14	168
5 x 2	10 ft.	8	80
5 x 2	12 ft.	10	120

Total linear feet of 6" x 2" = 408 ft.

of 4" x 3" = 384 ft.

of 5" x 2" = 200 ft.

Volume 6" x 2" : $408 \times \frac{6}{12} \times \frac{2}{12}$ = 34 cu. ft.

Volume 4" x 3" : $384 \times \frac{4}{12} \times \frac{3}{12}$ = 32 cu. ft.

Volume 5" x 2" : $200 \times \frac{5}{12} \times \frac{2}{12}$ = 13.9 cu. ft.

Ans. : 79.9 cu. ft.

CALCULATION OF LOG VOLUME

The problems involved in measuring log volumes will be dealt with in the Forest mensuration course. Only problems associated with the use of basic formulas will be considered here.

For the purposes of calculating volume logs are considered to have circular cross sections and to taper evenly from butt to crown. This is usually untrue but serves to give us a reasonable approximation of volume.

METHODS OF CALCULATING LOG VOLUME

There are four methods commonly used to calculate the volumes of logs.

These are :

1. The Mid Girth Method (a Huber's Method)
2. The Mean of Crown and Butt Girth Method (or Smalians Method)
3. The Sectional Method
4. The Form Factor Method

1. THE MID GIRTH METHOD

This method is also called Huber's method after the German forester who first recommended its use. This is the standard method for measuring log volumes in Western Australia.

The effect of log taper is overcome by making the measurement of log cross sectional area mid-way between the butt and crown end of the log. The assumption is that by measuring at this point, the average cross sectional area of the log is found.

Log Volume = SA at mid length x log length

If centre diameter is used this becomes

$$V = \frac{\pi D^2}{4} \times L$$

If centre girth is known the formula is

$$V = \frac{cG^2}{4\pi} \times L$$

(where cG = centre girth of log)

Example 1 :

A log 44' has a centre girth of 8'. Find its volume.

$$V = \frac{cG^2}{4\pi} \times L$$

$$= \frac{8 \times 8}{4} \times \frac{7}{22} \times 44$$

Ans. : 224 cubic ft.

Example 2 :

A log 21' long has a centre diameter of 2 ft. 9 in., find its volume.

$$V = \frac{\pi D^2}{4} \times L$$

$$V = \frac{22}{7} \times \frac{11}{4} \times \frac{11}{4} \times \frac{1}{4} \times 21$$

Ans. = 124.8 cu. ft.

2. THE MEAN OF CROWN AND BUTT GIRTH METHOD

This method is also called Smalians method, and is similar to the Mid-Girth method but instead of using the sectional area at mid length to overcome the effect of taper the mean of the sectional areas at the crown end and the butt end of the log is used.

This method is no more accurate than the Mid Girth method and requires an additional measurement and more complicated calculation. It is only used when :

(i) a gross irregularity (such as a hole or a swelling) at the mid point makes measurement there undesirable.

(ii) on quarterly assessment when logs have been removed but crown and stump remain.

It is important to note that the average of the sectional areas of the two ends should be used and not the sectional area equivalent to the mean of the two end girths.

There is very little error where the two end girths are nearly the same but the greater the difference between them the greater the error e.g. for a log with two end girths 36" and 18" the error is 10 per cent.

Log volume is given by

$$V = \frac{\text{SA at Crown} + \text{SA at Butt}}{2} \times \text{Length}$$

Example 3 :

Find the volume of a log 44 ft. long which has a crown girth of 8 ft. and a butt girth of 12 ft.

$$\text{SA of Crown} = \frac{(8)^2}{4\pi} = 5.1 \text{ sq. ft.}$$

$$\text{SA at Butt} = \frac{(12)^2}{4\pi} = 11.5 \text{ sq. ft.}$$

$$V = \frac{\text{SA at Crown} + \text{SA at Butt}}{2} \times \text{Length}$$

$$= \frac{(5.1 + 11.5)}{2} \times 44$$

$$= 365 \text{ cu. ft.}$$

3. THE SECTIONAL METHOD

In this method the log is considered to consist of sections of some constant and convenient length, in Australia usually 10 feet. Mid girth of each section is measured and volume is calculated by Huber's formula.

Usually logs are not an even 10 feet in length and the volume of the last log less than 10 feet, the "odd log" is calculated separately also by the mid girth method.

Since the volume of several short logs is calculated the volume measured should follow more closely the taper of the log and give a more accurate total volume measure.

Example 4 :

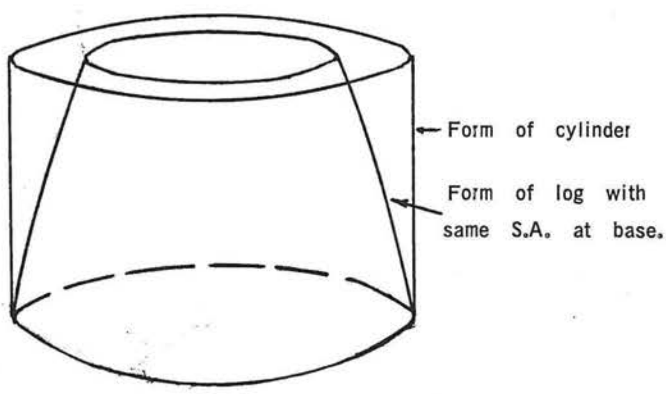
A log is 36 ft. long. Find its volume by the sectional method.

Log 1	Centre girth at 5'	$V = \frac{(cG)^2}{4\pi} \times 10 = a$
Log 2	Centre girth at 15'	$V = \frac{(cG)^2}{4\pi} \times 10 = b$
Log 3	Centre girth at 25'	$V = \frac{(cG)^2}{4\pi} \times 10 = c$
Odd log	Centre girth at 33'	$V = \frac{(cG)^2}{4\pi} \times 6 = d$

Volume of log = a + b + c + d

4. FORM FACTOR METHOD

The term form factor describes the ratio between the form of a cylinder and the form of a tapering log which has the same cross sectional area at its base as does the cylinder. The following diagram illustrates the relationship between these two figures.



Thus the volume of a log is given by

V = SA at butt x ff x length

Form factors are very variable and vary with species, genetic differences, type of stand etc. Average form factors of 0.8 for jarrah and 0.66 for karri are sometimes used.

Example 5 :

What is the volume of a 20 ft. jarrah log (ff = 0.8) with a butt girth of 9 ft. ?

$$V = \text{SA at butt} \times \text{ff} \times \text{length}$$
$$= \frac{9^2}{4} \times 0.8 \times 20$$
$$= 103.1 \text{ cu. ft.}$$

PROBLEMS

1. How many loads in 46,775 super feet?
2. How many cubic feet in 642 loads ?
3. How many super feet in the following order ?

3" x 2"	12 pieces	8 feet
6" x 2"	10 pieces	7 feet
8" x 1"	7 pieces	12 feet
3" x 2"	23 pieces	9 feet
$3\frac{1}{2}$" x $1\frac{1}{2}$"	10 pieces	7 feet

4. A log 39 ft. long has a centre girth of 6 ft. 10 in. Find its volume.
5. A log 28 ft. long has a centre diameter of 3'2". What is its volume?
6. A log 13 ft. long has a butt girth of 8'8" and a crown girth of 7'3". What is its volume in loads?
7. A log 43'6" is to be measured by the sectional method. At what distance from one end should the centre girth of the "odd log" be measured?
8. Find the volume by using the sectional method of a log 29'6" long for which the following data was collected:

   Centre girth at 5' = 9'5"
   Centre girth at 15' = 8'2"
   Centre girth at 24'9" = 7'6"

9. What is the volume of a Karri log (ff = 0.66), 106 ft. long with a butt girth of 26'3"?
10. A Jarrah log (ff = 0.8) has a butt diameter of 2'6" and a length of 33 ft. What is its volume in loads?

CHAPTER 17

SAWMILLING RECOVERY

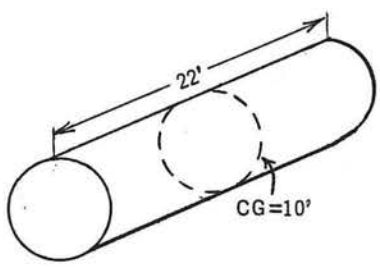
The aim of sawmilling is to produce useable sawn timber. Due to faults in the log, wastage in the form of sawdust and offcuts, and imperfect sawmilling technique, a large proportion of the wood material process is wasted. As a check on the milling technique and log quality it is customary to calculate the percentage timber recovery. If the "recovery percent" is lower than usual it can indicate :

1. Poor sawmill design, sawmilling method or very little attempt to recover small classes of useable timber.
2. Poor log quality.

The following examples illustrate the methods used to calculate mill and log recovery percent.

Example 1 :

A log is 22 feet long and has a centre girth of 10 feet. The volume of timber recovered is 100 cubic feet. What is the sawmilling recovery?



Step 1 :

FORMULA : Log Volume $= \frac{(cG)^2}{4\pi} \times L$

$= \frac{10}{4} \times \frac{10}{22} \times \frac{7}{1} \times \frac{22}{1}$ cu. ft.

Volume = 175 cubic feet.

Step 2 :

FORMULA : Percent = Fraction x 100

Substituting $= \frac{100}{175} \times 100\%$

Ans. : Recovery = 57.1%

Example 2 :

The general recovery in a particular mill is 33.1/3%. In a day's run 35 loads of logs are processed. What volume of sawn timber (in cubic feet) could be expected from the day's cutting?

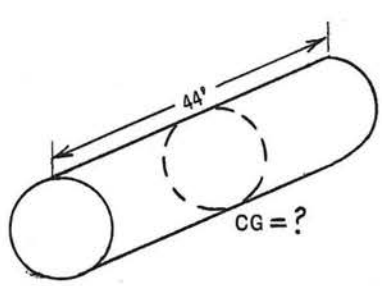
Step 1 :

FORMULA : Percent = Fraction x 100

Substituting : $33.1/3 = \frac{X}{35} \times 100$

$X = \frac{100}{3} \times 35 \times \frac{1}{100}$

$X = \frac{35}{3}$ loads



Step 2 :

1 load = 50 cubic feet

$\frac{35}{3}$ loads = X

$X = 50 \times \frac{35}{3}$ cubic feet

$= \frac{1750}{3}$ cubic feet

Ans. : Recovery = 583.1/3 cubic feet.

Example 3 :

The recovery from a 44 foot log is 100 cubic feet. This represents a 40% of the original volume. What was the centre girth of the log?

Step 1 : For full log volume.

FORMULA : Percent = Fraction x 100

Substituting : $40 = \frac{100}{X} \times \frac{100}{1}$

Log Volume = 250 cubic feet.

Step 2 :

FORMULA : $LV = \frac{(cG)^2}{4\pi} \times \text{Length}$

Substituting : $250 + (cG)^2 \times \frac{7}{88} \times 44$

$(cG)^2 = 250 \times \frac{2}{7}$

$cG = \sqrt{71.4285}$

$= 8.45$ feet.

Ans. : Centre Girth = 8.45 feet or 8'5.4''

PROBLEMS

1. What is the percentage recovery from a 40 cubic foot log when 16 cubic feet is in off-cuts and 8 cubic feet in sawdust?
2. From a log of 30 feet lenth and 7 feet centre girth 80 cubic feet are recovered as sawn timber. What % recovery has been achieved?
3. A mill has the following intake and recovery over a 5 day period. What is the average recovery percent?

	Log Volume	Timber Recovery
Day 1	1655 cubic feet	810 cubic feet
Day 2	1720 " "	512 " "
Day 3	1828 " "	788 " "
Day 4	1558 " "	725 " "
Day 5	1850 " "	877 " "

4. A 45% recovery from a log 25 feet long is 53 cubic feet. What is the centre diameter of the log?

CHAPTER 18

JOB COSTING

1. DEPARTMENTAL JOB COSTING

At times it is desirable to find out what it costs to do a particular job. Interest for example may be in the cost of a thinning operation, the average cost of installing culverts, the cost per load in the square of a pine mills operation.

The total cost of a job may be made up from some or all of the following factors.

(a) Cost of labour

(b) Cost of machinery e.g. equipment and mileage

(c) Cost of materials e.g. hardware, hormones for poisoning etc.

(d) Overheads and administration costs

Sometimes individual costs of items a-c are useful but when quoted should be qualified. For example, "cost of labour without overheads" or "cost per acre of equipment used". Without these qualifications these figures would be meaningless and at times deceptive.

Overhead and Administration costs require closer definition.

OVERHEADS

Overheads are amounts additional to wages charged against each job. This addition is necessary to cover the cost of leave, pay roll tax, workers compensation, tool allowances, etc.

Overheads are calculated by summing the total wages expenditure and adding 25% of the total.

ADMINISTRATION

This addition is necessary to cover the costs of administration. It covers the salaries of local officers and head office staff.

Administration costs are calculated by summing wages plus overheads, materials, equipment and vehicle costs and adding 10% to the total.

Once the total cost of the job has been calculated the amount of work done must be assessed. For example, how many acres, loads of timber, miles of road, etc. have been completed or produced? The cost can then be quoted on a per unit basis.

Example 1 :

The costs involved in grading 10 miles of road were as follows :

Wages	\$25.00
Machinery	\$84.00

What is the cost per mile ?

Wages	= \$25.00
Overheads at 25% of wages	= 6.25
Wages + Overheads	= 31.25
Machinery	= 84.00
Total	\$115.25
Administration at 10%	11.52
Total cost of job	\$126.77

Cost per mile $\frac{\$126.77}{10 \text{ miles}} = \$12.68$

Example 2 :

The men, machines and materials required to put in 22 culvert pipes 24 ft. long, 15" in diameter were as follows :

Wages	1 overseer 4 days at $50 per week 1 forest workman 4 days at $40 per week 1 back hoe driver 4 days at $48 per week 1 dozer driver 1 day at $55 per week
Materials	66, 8 ft. x 15" concrete pipes at $7 each
Vehicles and Equipment	7 ton truck, 4 trips, 30 miles return at 20c. per mile 1 trip 50 mile return at 20c. per mile 5 ton gang truck 2 trips, 30 miles return at 16c. per mile Back hoe, 25 hours at $6 per hour D4 bulldozer, 6 hours at $15 per hour Low loader, 1 trip, 30 miles return at 25c. per mile

What is the cost per culvert?

Calculation

Wages	Overseer 4 days @ $50/week	=	$ 40.00
	Forest workman 4 days @ $40/week	=	$ 32.00
	Back hoe driver 4 days @ $48/week	=	$ 38.40
	Dozer driver 1 day @ $55/week	=	$ 11.00
			$121.40
	Overheads at 25%	=	30.35
	Wages + Overheads	=	$151.75
Materials	66 pipes @ $7.00 each	=	$462.00
Vehicles and Equipment	7 ton truck 170 miles @ 20c./mile	=	$ 34.00
	5 ton truck 60 miles @ 16c./mile	=	$ 9.60
	Back hoe 25 hours @ $6/hour	=	$150.00
	D4 bulldozer 6 hours @ $15/hour	=	$ 90.00
	Low loader 30 miles @ 25c./mile	=	$ 7.50
	TOTAL		$291.10

Total cost is :

Wages + Overheads	=	$151.75
Materials	=	$462.00
Vehicles and Equipment	=	$291.10
		$904.85
Administration at 10%	=	$ 90.48
Total Cost		$915.33

Unit cost per culvert = $\frac{\$915.3}{22 \text{ culverts}}$ = $41.61

2. WORK STUDIES

Job costing is important in carrying out work studies.

If the cost of the unit output of workers under different conditions is calculated the efficiency of various work methods and tools can be calculated. Studies of the normal work capacity of each man are also essential in calculating fair rates of pay per piece workers. When carrying out work studies care must be taken to ensure that good relations with the men are maintained and that they understand why the study is being done.

Example 1 :

Compare the efficiency of thinning jarrah coppice by (a) falling and (b) poisoning.

	Falling	Poisoning
Area covered per man per day	2.5 ac.	3.5 ac.
Wages per man per day incl. overheads	$10	$10
Cost of materials and equipment	$ 6	$13
Total cost	$16	$23
Cost per acre =	$6.40	$6.58

Thus the thinning of coppice by falling is the cheapest method.

3. COSTING OF EQUIPMENT

Often it is desirable to compare the efficiency of two machines by calculating the cost of their operation per unit of work done. The cost of machinery can be

(i) Capital costs and depreciation
(ii) Repair and maintenance costs
(iii) Fuel costs
(iv) Miscellaneous costs, licences, accident insurances etc.

While the costs (ii) to (iv) above are reasonably easy to calculate, there are various ways of calculating capital costs and depreciation and the advantages of each is the subject of economic theory. While in the examples below this factor may be taken into account no attempt is made to discuss the best methods of dealing with this problem.

Once the total costs have been derived, unit cost per mile or per hour can be calculated.

Example 1 :

The following costs were incurred in the operation of a utility over 10,300 miles.

(i) Depreciation from $2,000 to $1,600 (exclude the cost of interest on capital)

(ii) Repairs and Maintenance : Parts and materials $113.60
Labour $ 60.00

(iii) Fuel consumed 410 galls. at 30c. /gall.
Oil consumed 11 galls. at $1.60/gall.

(iv) Insurance costs $50.

What are the running costs per mile?

Calculation

Depreciation	=	$400.00
Parts and Materials	=	$113.60
Labour	=	$ 60.00
Fuel	=	$123.00
Oil	=	$ 17.60
Insurance	=	$ 50.00
Total		$764.20

Cost per mile = $\frac{\$764.20}{10{,}300 \text{ miles}}$ = 7.4 cents/mile.

Example 2 :

A study was carried out comparing the economics of using two sizes of bulldozer on a road building job. Which machine was the cheapest?

	D4	D7
Volume of earth removed in 1 week	7,000 cu. yds.	12,000 cu. yds.
Capital cost (to be depreciated at 20% p.a.)	$40,000	$65,000
Maintenance costs	$12.00	$12.00
Fuel consumption at 20c. /gall.	320 galls.	480 galls.
Insurance costs per year	$200	$300

Calculation

D4

Capital cost per week $40,000 \times \frac{20}{100} \times \frac{1}{52}$ = $153

Maintenance costs = 12

Fuel costs $\frac{20}{100} \times 320$ = 64

Insurance costs per week = $\frac{1}{52} \times 200$ = 4

$233

Cost per cubic yard = $\frac{\$233}{7,000 \text{ cu. yds.}}$ = 3.3 cents/cu. yd.

D7

Capital cost per week = $65,000 \times \frac{20}{100} \times \frac{1}{52}$ = $250

Maintenance costs = 12

Fuel costs $\frac{20}{100} \times 480$ = 96

Insurance costs per week = $\frac{1}{52} \times 300$ = 6

$364

Cost per cubic yard = $\frac{\$364}{12,000 \text{ cu. yds.}}$ = 3.0 cents/cu. yd.

The cost per cubic yard using the D7 machine is less.

PROBLEMS

1. A nursery shade house took a carpenter, earning $50.00 per week, 6 days to build and he used materials valued $343.00. What is the total cost of the building to the Forests Department?

2. The following costs were incurred in planting a 500 acre compartment with pines.

Wages	1 overseer 3 weeks at $56 per week 10 workmen 3 weeks at $46 per week
Materials	300,000 pines at $2.50 per thousand
Vehicles	Gang truck 1 return trip of 20 miles each day for 3 weeks at 15 cents/mile. 1 plant delivery each day for 3 weeks, 80 miles round trip at 16 cents/mile.

What is the total cost of this planting per acre to the department?

3. $8,000 is allocated to the construction of a 6 mile section of the road. So far construction costs have been as follows :

Clearing	$2,000
Forming	3,000
Culverts	1,200

So far 1 mile has been gravelled in 2 days by 4 trucks carrying approximately 25 loads per day each and the average round trip distance is 8 miles. If it is expected that final grading will cost $300 for the whole road, (and this must be done), how many more days can gravel be carted before the money allocated runs out if the cost per mile for each truck is 20c. ? How many miles of road will be completed?

4. Two chainsaw makes are tested over a period of years, both working approximately 200 hours per year. The following facts and figures were recorded :

	Saw A	Saw B
Capital cost	$300	$200
Life of saw	4 yrs.	3 yrs.
Cost of repairs	$70	$40
Number of chains used	35	28
Cost of chains	$20	$18
Fuel consumed @ 30c. /gall.	500 galls.	400 galls.

Assume that the saws were both worth $20 at the end of their life, which would be the most economical saw to use?

5. The following figures were recorded of the cost of constructing culverts by 2 methods :

A. Gang Labour and Crane

Output per day 3 culverts

Labour costs 6 men at an average $48 per week

Crane costs $2.80 per day

B. Back Hoe and Crane

Output per day 5 culverts

Labour costs 2 men at an average $50 per week

Back hoe costs $4 per hour and works 8 hours per day

Crane costs $2.80 per day

Which would be the cheapest method?

6. An overhead telephone line will last 20 years before renewal, costs $100 per mile to build but costs $60.00 per mile per year to maintain. An underground telephone line will last 60 years before renewal, cost $2,000 per mile to lay and costs $10.00 per year to maintain. Which type of line is the cheapest?

CHAPTER 19

FOREST ENGINEERING PROBLEMS

Most of the problems encountered in forest engineering are simple arithmetical calculations. Some problems will also be dealt with in the surveying and engineering course. Two sections however require closer attention. Before attempting this section get your D.F.O. to show you how to use tangent tables.

Grade

The grade of a road or any other feature for that matter (drains, river beds etc.) can be expressed in the following ways.

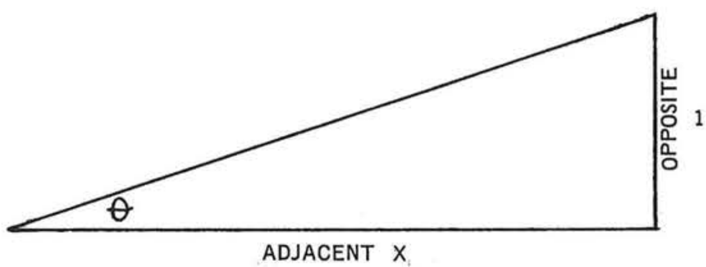
(i) The number of degrees from horizontal (i.e. a 6^{o} slope).

(ii) As the ratio of movement between vertical and horizontal distance (i.e. 1 in 8).

(iii) The vertical rise for a fixed horizontal distance. (i.e. 18" in the chain). This expression of grade is often given by the percentum scale on instruments which records the number of units vertical rise for 100 horizontal units.

It is often useful to be able to convert one expression of grade to another.

A. CONVERSION OF DEGREES TO RATIO MEASUREMENT



Usually the vertical side of the ratio is given unity (i.e. 1 in 4 or 1 in 10)

Since $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent sides}}$

$$\tan\theta = \frac{1}{X}$$

$$X = \frac{1}{\tan\theta}$$

Therefore the ratio of grade (1 in x) is the tangent of the number of degrees from horizontal divided into one.

Example 1 :

A road is selected at 6^{o} fall. What is the grade ratio?

$$\text{Grade ratio (X)} = \frac{1}{\tan 6^{o}}$$

$$= \frac{1}{0.1051}$$

i.e. grade is 1 in 9.52

Example 2 :

A road is to be selected on a grade of 1 in 10. What will be the fall in degrees on which the abney can be set?

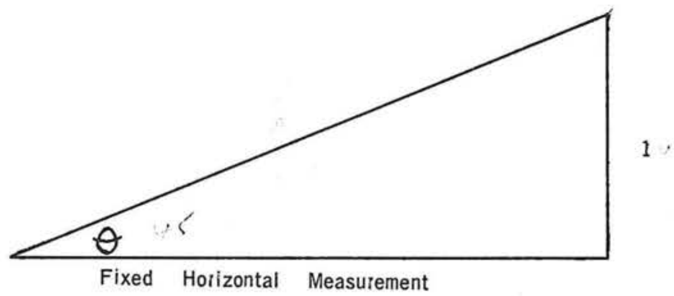
$$\text{Grade ratio (X)} = \frac{1}{\tan\theta}$$

$$10 = \frac{1}{\tan\theta}$$

$$\tan\theta = 0.1$$

$$= 5^{\circ}43'$$

B. CONVERSION OF RATIO TO RISE PER UNIT OF HORIZONTAL DISTANCE MEASUREMENT



Since grade ratio is expressed as 1 vertical unit per X horizontal units and grade measurement by using a rise per fixed horizontal measurement is expressed

as X vertical units per 1 horizontal unit

Conversion from one to the other can be given by

$$\frac{1}{\text{horizontal ratio}} = \frac{\text{vertical units}}{\text{fixed horizontal measurement}}$$

Example 1 :

How many inches per chain must a road rise to have a grade of 1 in 44.

$$\frac{1}{44} = \frac{X \text{ ins.}}{66 \text{ ft.}}$$

$$\frac{X}{12} = \frac{66}{44} \text{ ins.}$$

$$X = \frac{66}{44} \times 12$$

$$X = 18''$$

Example 2 :

A road falls at a constant grade by 200 ft. in a mile. What is the grade ratio?

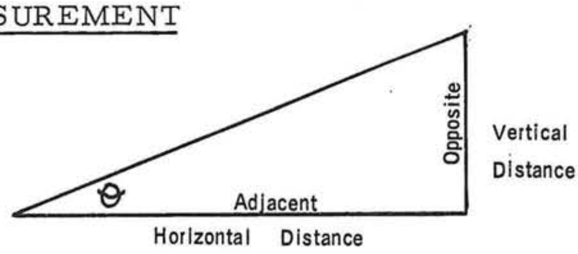
$$\frac{1}{X} = \frac{200}{5280}$$

$$X = \frac{5280}{200}$$

$$X = 26.4$$

Ans. : 1 in 26.4

C. CONVERSION OF DEGREES TO RISE PER UNIT OF HORIZONTAL DISTANCE MEASUREMENT



Since the horizontal distance is constant and

$$\text{Tan}\,\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The relation between the two expressions can be given by

$$\text{Tan}\,\theta = \frac{\text{vertical distance}}{\text{constant horizontal distance}}$$

Example 1 :

How many links per chain does a grade drain at 10^{o} depression fall?

$$\text{Tan}\,10^{o} = 0.1763$$

$$0.1763 = \frac{X}{100 \text{ links}}$$

$$X = 0.1763 \times 100$$

$$= 17.6 \text{ links.}$$

Example 2 :

A stream bed falls 120 ft. in every mile. What is the grade in degrees?

$$\text{Tan}\,\theta = \frac{12}{5280}$$

$$\text{Tan}\,\theta = 0.0227$$

$$\theta = 1^{o}18'$$

PROBLEMS

1. Instructions are given to construct a road on a maximum grade of 1 in 9. The abney level however only has a scale which reads in degrees from the horizontal. What is the maximum elevation in degrees?
2. It is desired to construct a road from the top of a hill (800 ft. above sea level) to a bridge crossing (400 ft. above sea level) 1.5 miles away. On what constant grade ratio can the road location be selected?
3. A truck can just climb a 9^{o} slope. Can it negotiate a road that rises 10 ft. in every chain?
4. A new railway has a constant grade of 1 in 200. How many feet per mile does it rise?
5. A roadside embankment is three vertical to two horizontal units. What is the slope in degrees from the horizontal?

CALCULATION OF THE VOLUME OF CUT AND FILL

When a road is constructed in hilly country it is often necessary to cut into a hillside or through a spur and fill in gullies or depressions to obtain an easier grade. Since the cost of these earthworks may be quite high it is usually desirable to make an estimate of the volume of earth to be removed when planning the location of a road.

This is most conveniently found by surveying the level of the existing surface from a reference point and plotting it in side elevation to scale on graph paper and then inserting the desired surface level of the proposed road.

If the slope of the ground at right angles to the direction of the road is measured at regular intervals then the cross section of the excavation or fill at these points can be drawn. The average cross section of the excavation or fill multiplied by the length of the section will give the volume.

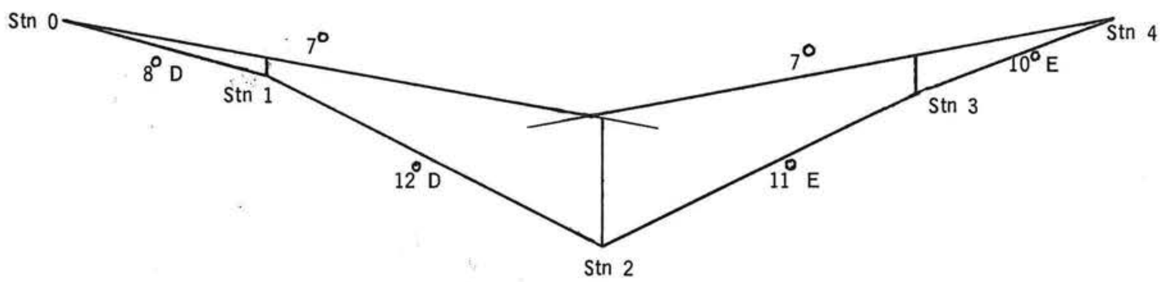
Example 1 :

From a reference point the survey of levels across a gully were :

Stn. No.	Dist. from Ref. Point	Slope from last Stn.	Slope to left of Rd.	Slope to rt of Road
1	100 links	8° Depression	6° E	9°D
2	200 links	12° Depression	10° E	10°D
3	300 links	11° Elevation	12° E	10°D
4	400 links	10° Elevation		

It is desired to construct a road across the gully at a grade of not more than 1 in 8 (7°) from Station 0 to 4. How much fill would be required if the road is to be 20 ft. wide and the embankment slope 1 in 1?

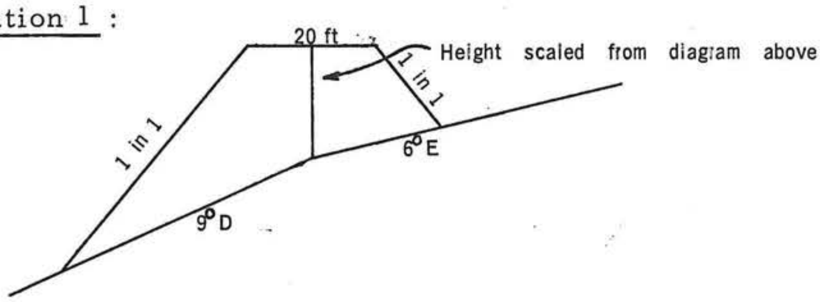
Plot the longitudinal section on graph paper to scale as follows :



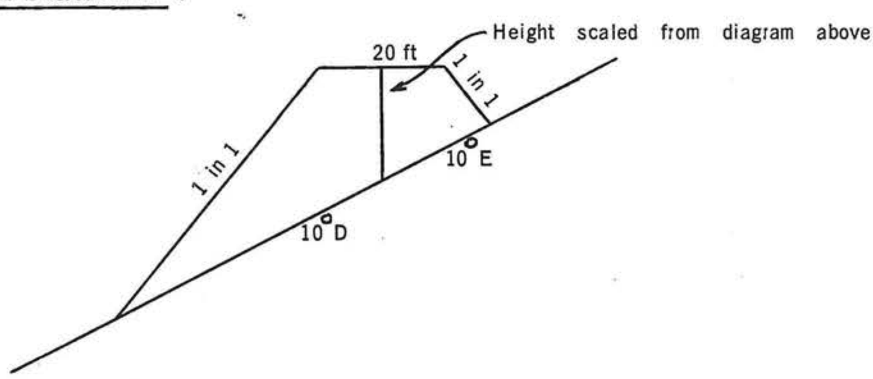
Draw two lines from the two end points (Stations 0 to 4) at the required grade 7° (or 1 in 8) and these will intersect.

Now scale off the heights of the proposed road, above the existing surface at Stations 1, 2 and 3 and draw cross sections of the road at these points to scale.

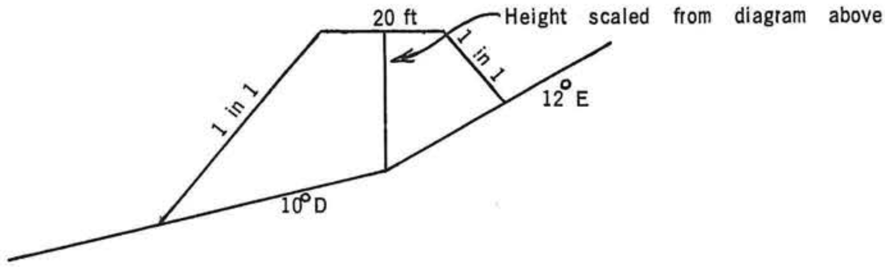
Cross Section Station 1 :



Cross Section Station 2 :



Cross Section Station 3 :



Find the area of the cross section of fill at each of these points.

Example Station	Cross Sect. Area
1	79 sq. ft.
2	283 sq. ft.
3	164 sq. ft.

Then the average cross sectional area for each section of the fill is calculated and multiplied by the length of the section to give volume. The sum of these volumes is the total volume of the fill.

Section	Area of fill one end	Area of fill other end	Average	Length of Section	Volume
0 - 1	0	79 sq. ft.	39 sq. ft.	66	2,574
1 - 2	79 sq. ft.	283 sq. ft.	181 sq. ft.	66	11,946
2 - 3	283 sq. ft.	164 sq. ft.	223 sq. ft.	66	15,718
3 - 4	164 sq. ft.	0 sq. ft.	82 sq. ft.	66	5,412
					35,650 cu. ft.

Ans. : 35,650 cu. ft.

PROBLEMS

1. The following is the data for a survey across a spur from a known reference point (Station 0)

Stn.	Dist. from Stn. 0	Slope	Slope to left of Road	Slope to right of Road
1	100 ft.	7°E	12°E	10°D
2	200 ft.	9°E	8°E	4°D
3	300 ft.	10°E	7°E	8°D
4	400 ft.	8°E	16°E	17°D
5	500 ft.	4°D	12°E	10°D
6	600 ft.	12°D	9°E	8°D
7	700 ft.	10°D	17°E	16°D
8	800 ft.	9°D		

How much material must be removed to construct a road with a maximum grade of 6°, 20 ft. wide with batter slopes 1 in 1.

2. It is proposed to construct a road across a gully having a grade not more than 1 in 10, 25 ft. wide having embankment slopes 2 in 1. The following is the appropriate survey data.

Stn.	Dist. from Stn. 0	Slope	Slope to left of Road	Slope to Rt. of Road
1	2 chn.	7^{o}D	0^{o}	0^{o}
2	4 chn.	6^{o}D	2^{o}E	1^{o}D
3	6 ch.	12^{o}D	4^{o}E	2^{o}D
4	8 chn.	level	2^{o}E	0^{o}D
5	10 chan.	12^{o}E	1^{o}E	5^{o}D
6	12 chn.	14^{o}E	2^{o}E	8^{o}D

CHAPTER 20

PRESENTATION OF DATA

Many purposes in forestry require masses of data to be collected, for example the girths or height of trees in a plot.

Take the example where a sample plot in a pine plantation has been measured for d. b. h. We can present this data of the individual tree diameters as a mass of measurements.

i. e. D.B.H. in inches

7.9"	8.6"	9.4"	9.7"
11.3	10.2	7.2	10.1
9.8	11.1	11.7	7.9

This method is cumbersome, especially if there are a large number of trees in the plot. It is also hard for the person who is looking at the data to gather up in his mind and see what is the average size tree and how much variation there is in tree d. b. h. There are many ways of presenting this data in a clear and simple manner. A few are :

(1) FREQUENCY TABLE

The data is grouped into size classes

e. g.

d. b. h. class (ins.)	frequency
6.5 - 7.4	1
7.5 - 8.4	2
8.5 - 9.4	2
9.5 - 10.4	4
10.5 - 11.4	2
11.5 - 12.4	1
	10

In this case the class interval, the difference between the bottom of one class and the top measurement of the same class is 1.0". The class interval can be of any size depending upon the whim of the person compiling the data and will vary with the types and number of measurements to be grouped. For example the heights of trees in a karri plot may be grouped in the classes.

ht. in ft.

80 - 90

90 - 100 here the class interval is 10'

100 - 110

110 - 120

120 - 130

In grouping data the problem arises when measurements fall on the boundary between one class and the next, as to which class the measurement belongs.

For example - take the classes.

d. b. h. in inches

6.5 - 7.5

7.5 - 8.5

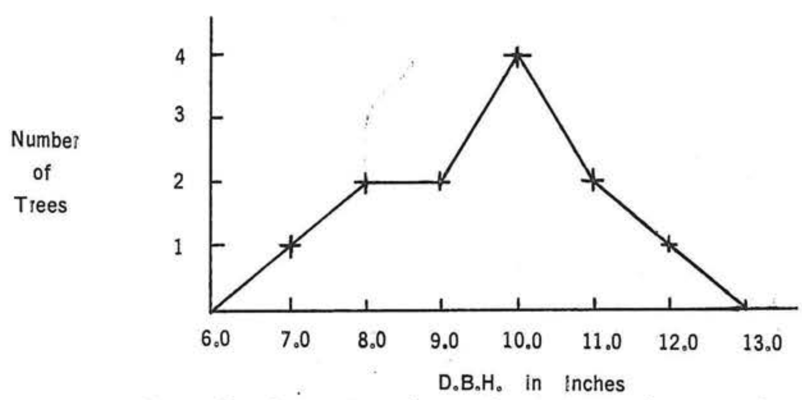
What do we do with a measurement of 7.5"?

There are two alternatives.

(i) Take the limits of the class to one more decimal place than the data which we are grouping, i.e.
6.50 - 7.49
7.50 - 8.49
Here a measurement of 7.5 obviously belongs to the second class.

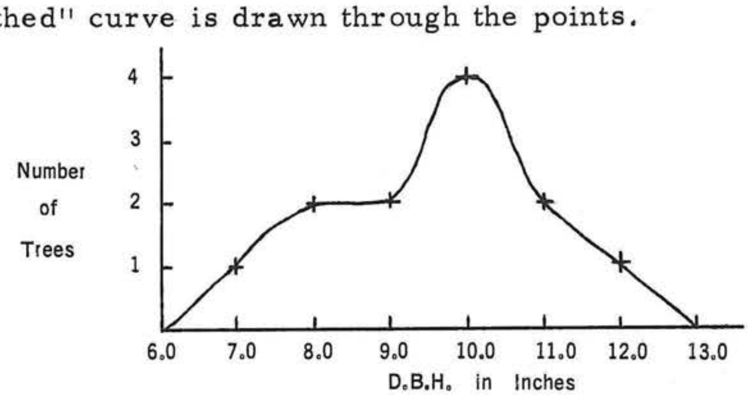
(ii) This is usually abbreviated to making the class limits.
6.5 - 7.4
7.5 - 8.4
and it is understood that the first class includes measurements from 7.40 to 7.49.
In assigning class limits it is usual to select them so that the centre of the class is a whole or round number, i.e. in the above example the centres of the classes are 7.0 and 8.0
In selecting height classes we may decide on 85 - 95
95 - 105 etc.
so that the centre of the class is 90 and 100 ft. respectively.
This is done to make the data easier to plot.

(iii) Straight line graph (Frequency Polygon)
Once we have compiled a frequency table we can present this data in graphical form. In the example given we can plot :



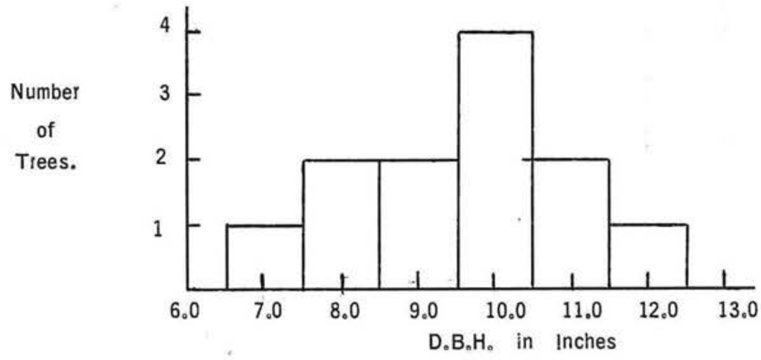
The centre of each class is plotted against the number of observations (in this case trees) in that class.
In presenting data in graphical form the vertical axis of the graph is always the variable axis. In this case for each d.b.h. class the number of trees will vary, therefore since the vertical axis is the variable axis the number of trees is plotted along it.

(iv) Curve Graph (Frequency Curve)
From the frequency table the mid point of the class is plotted against the frequency but instead of joining the points with a straight line, a "smoothed" curve is drawn through the points.



(v) Histograms

A histogram is a pictorial representation of the data contained in the frequency table. It can resemble the form of graphical representation, i.e.



PROBLEMS

1. Group the following tree heights into a frequency table with class intervals of 5 feet.

Height in Feet

62	79	82	57	63	66	74	58
78	77	68	84	64	69	73	70
75	72	63	65	80	74	66	77
75	71	73	69	62	77	70	84
66	71	75	79	68	72	75	78

2. Draw a smoothed graph (frequency curve) for the above data from the frequency table you have compiled.

3. Draw a histogram of the following data. Decide your own class intervals.

dbh of Trees in a Plot

6.3	7.5	14.4	15.9	12.9
10.8	9.7	8.8	11.7	13.9
12.2	10.6	13.3	14.2	10.1
8.9	10.3	10.9	11.7	11.4
11.8	12.2	11.0	10.0	12.1

4. Plot the data of Q3. on a straight line graph (determine your own class intervals).

APPENDIX 1

LINEAR MEASURE

The following are commonly used units of lineal measure.

British Units	Metric Units	Mixed Equivalent Units
1 inch	2.540 centimetres	
1 foot	30.480 centimetres	12 inches
1 foot		1.515 links
1 yard	91.440 centimetres	3 feet
1 chain	20.117 metres	22 yards
1 furlong		10 chains
1 mile	1.60935 kilometres	8 furlongs
1 mile		1760 yards
1 mile		5280 feet
1 link	0.20117 metres	7.92 inches
1 chain	20.117 metres	100 links
0.3937 inches	1 centimetre	10 millimetres
	1 decimetre	10 centimetres
39.37 inches	1 metre	10 decimetres
	1 decametre	10 metres
	1 hectametre	10 decametres
0.62137 miles	1 kilometre	10 hectametres
1093.611 yards	1 kilometre	

APPENDIX 2

SQUARE MEASURE

The following are commonly used units of square measure.

British Units	Metric Units	Mixed Equivalent Units
1 sq. inch	6.4516 sq. centimtrs.	
1 sq. foot	0.0929 sq. metres	144 sq. inches
1 sq. yard	0.8361 sq. metres	9 sq. feet
1 sq. chain	404.687 sq. metres	484 sq. yards
1 acre	0.404685 hectares	10 sq. chains
1 acre	4046.85 sq. metres	4840 sq. yards
1 acre		43560 sq. feet
1 sq. mile	2.590 sq. kilometres	640 acres
1 sq. mile	259 hectares	
0.1550 sq. inches	1 sq. centimetre	
10.7639 sq. feet	1 sq. metre	10,000 sq. centimetres
2.4711 acres	1 hectare	10,000 sq. metres
0.38610 sq. miles	1 sq. kilometre	100 hectares
247.106 acres	1 sq. kilometre	

APPENDIX 3

CUBIC AND LIQUID MEASURE

The following are commonly used units of cubic and liquid measure.

British Units	Metric Units	Mixed Equivalent Units
1 cubic inch	16.38703 cubic centimetres	
1 cubic foot	0.02832 cubic metres	1728 cubic inches
1 cubic yard	0.76455 cubic metres	27 cubic feet
1 cubic chain		10648 cubic yards
1 acre foot		272250 gallons
1 cubic mile		3379200 acre feet
1 pint	0.56825 litres	
1 quart	1.13650 litres	2 pints
1 gallon	4.5460 litres	8 pints
1 cubic foot		6.25 gallons
1 super foot		0.08333 cubic feet
1 cubic foot		12 super feet
1 load	1.4158 cubic metres	50 cubic feet
1 cord	3.625 cubic metres	128 cubic feet
0.061023 cubic inches	1 cubic centimetre	1000 cubic millimetres
1.30795 cubic yards	1 cubic metre	1,000,000 cubic centimetres (approx.)
	1 millimetre	1 cubic centimetre
1.7598 pints	1 litre	1000 millilitres

APPENDIX 4

WEIGHT MEASURE AND EQUIVALENT CUBIC MEASURE

The following are commonly used units of weight and some cubic equivalents.

British Units	Metric Units	Mixed Equivalent Units	Equivalent Cubic Units
1 ounce	28.3495 gms		
1 pound	0.453592 kilograms	16 ounces	
1 stone		14 pounds	
1 quarter		2 stone	
1 hundred weight		8 stone	
1 " "		112 pounds	
1 ton	1016.047 kilograms	20 hundred-weight	
1 ton	1.016047 metric tons	2240 pounds	
	1 gram		1 cubic centimetre
2.2046 pounds	1 kilogram	1000 grams	1 litre (approx.)
		20 ounces (app.)	1 pint (fresh water
		10 pounds (app.)	1 gallon " "
		62.5 pounds (app.)	1 cubic foot "